Right Triangles

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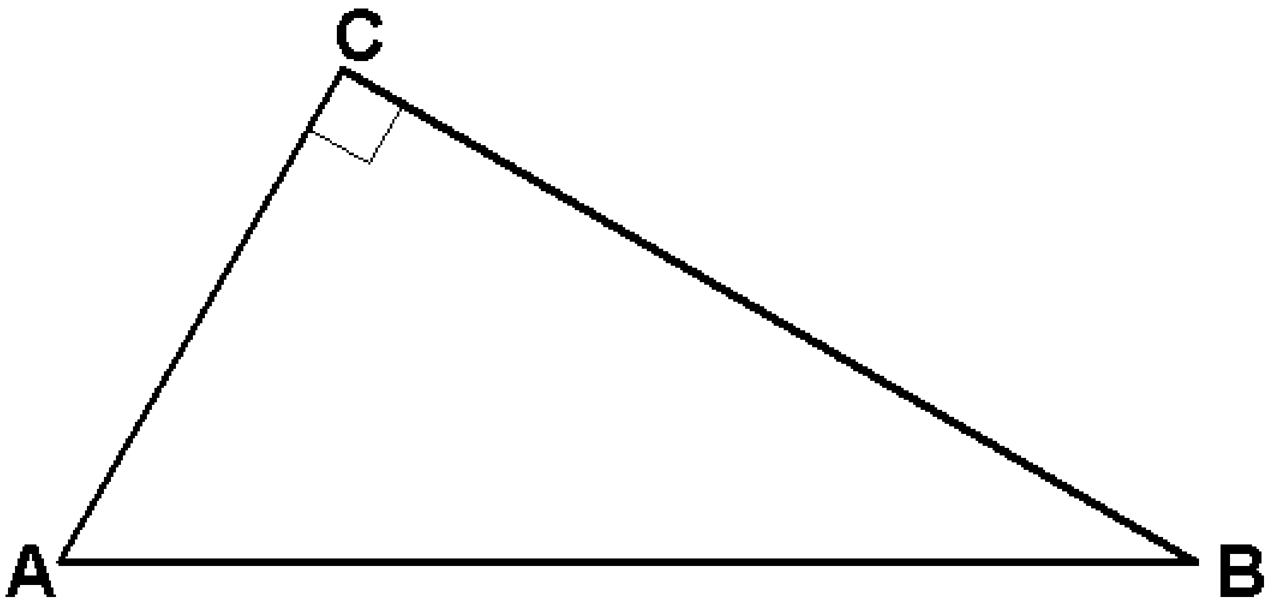
Consider the Right Triangle.



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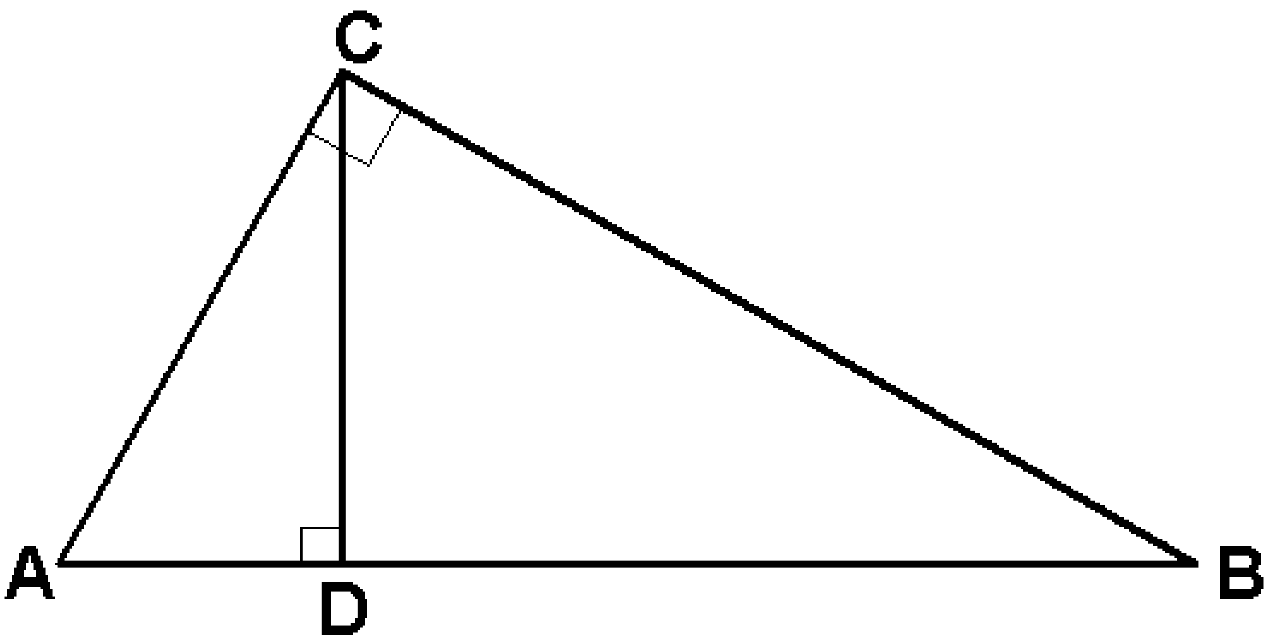
If we draw a vertical line from vertex C to a point D on our base $\overline{AB}$, we form other right triangles.



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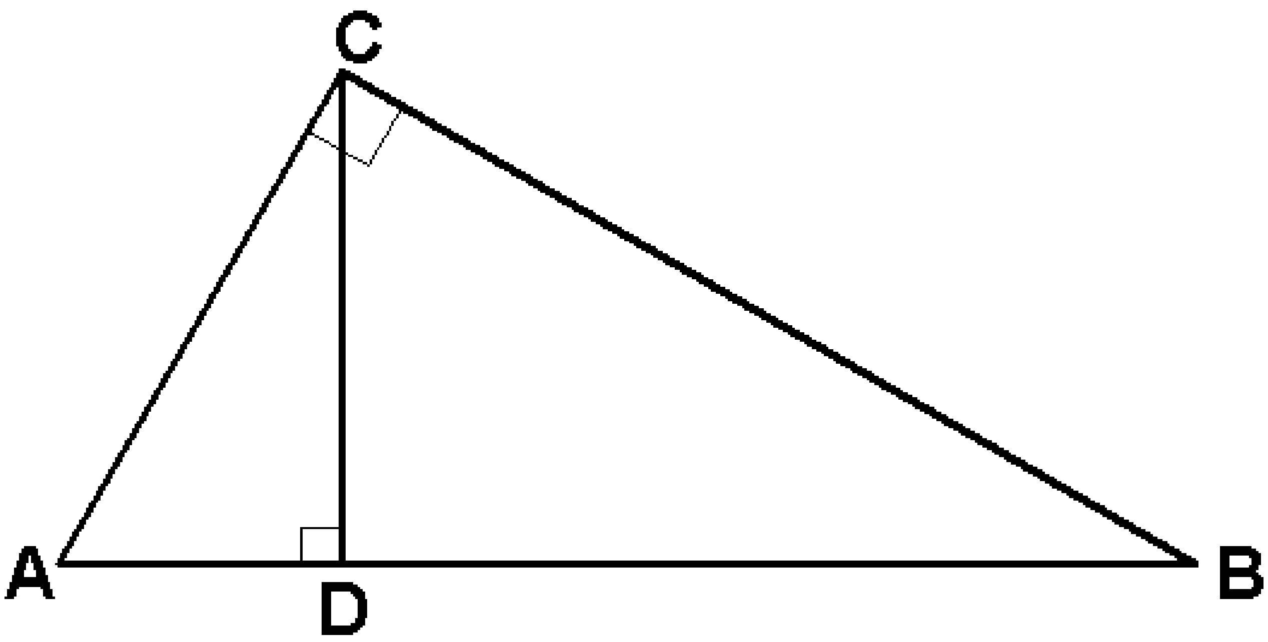
We have now created three right triangles $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$. These triangles are all similar!



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Recall that similar triangles have congruent (equal measure) corresponding angles.

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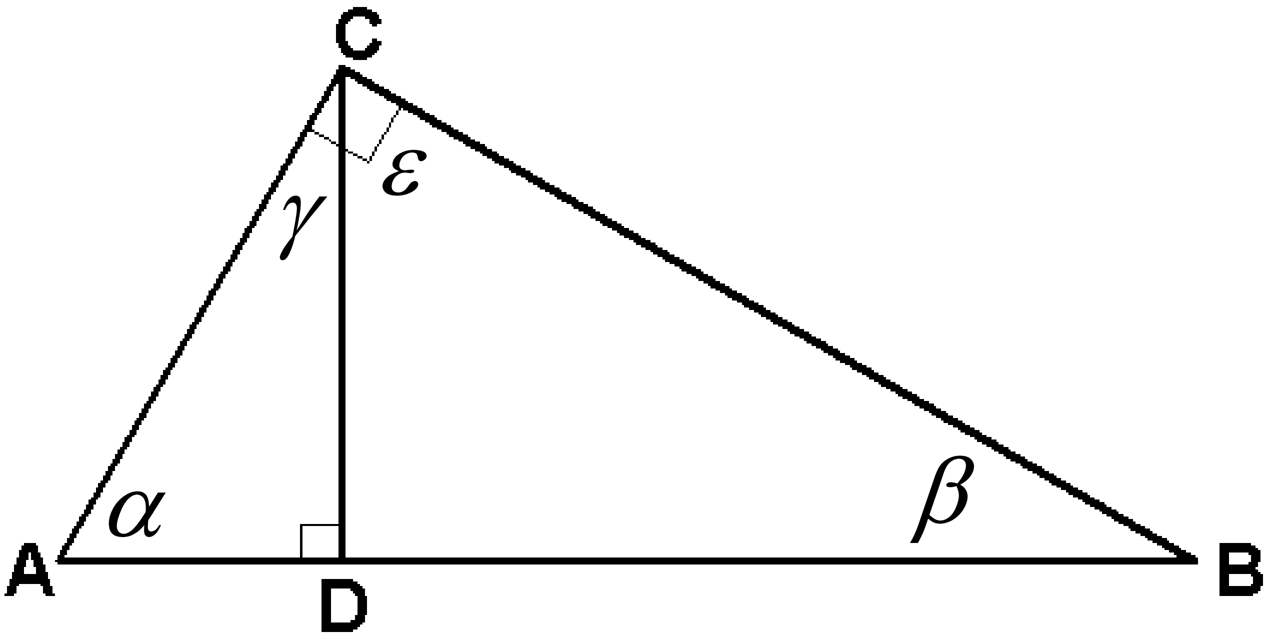
Claim: $\angle\varepsilon = \angle\alpha,\ \angle\gamma = \angle\beta$



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We know the following

$$\angle\gamma + \angle\varepsilon = 90^\circ$$

but, $\angle\alpha + \angle\gamma = 90^\circ$ as well

so, $\cancel{\angle\gamma} + \angle\varepsilon = \angle\alpha + \cancel{\angle\gamma}$

$$\rightarrow \angle\varepsilon = \angle\alpha$$

Similarly,

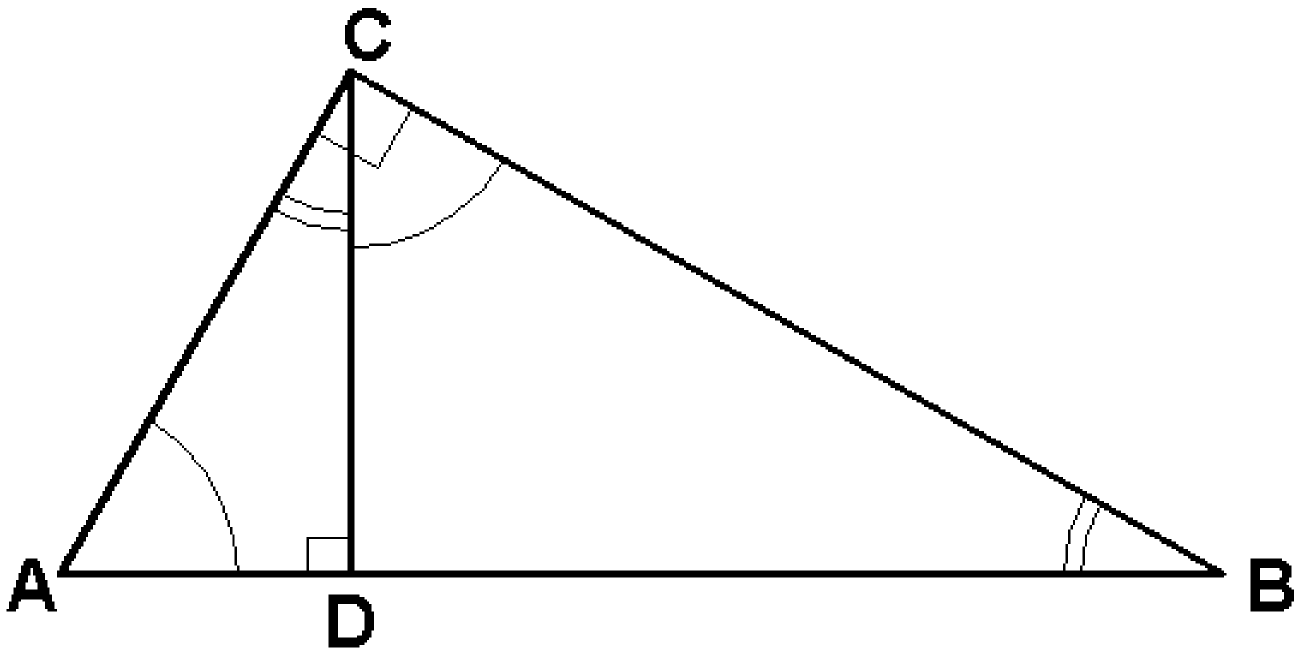
$$\angle\gamma + \angle\varepsilon = 90^\circ$$

but, $\angle\varepsilon + \angle\beta = 90^\circ$

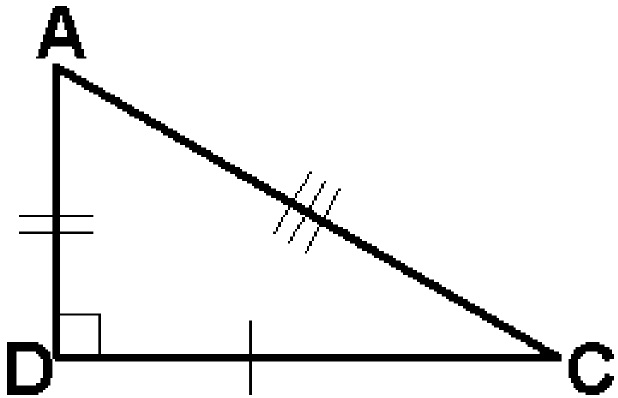
$$\angle\gamma + \cancel{\angle\varepsilon} = \cancel{\angle\varepsilon} + \angle\beta$$

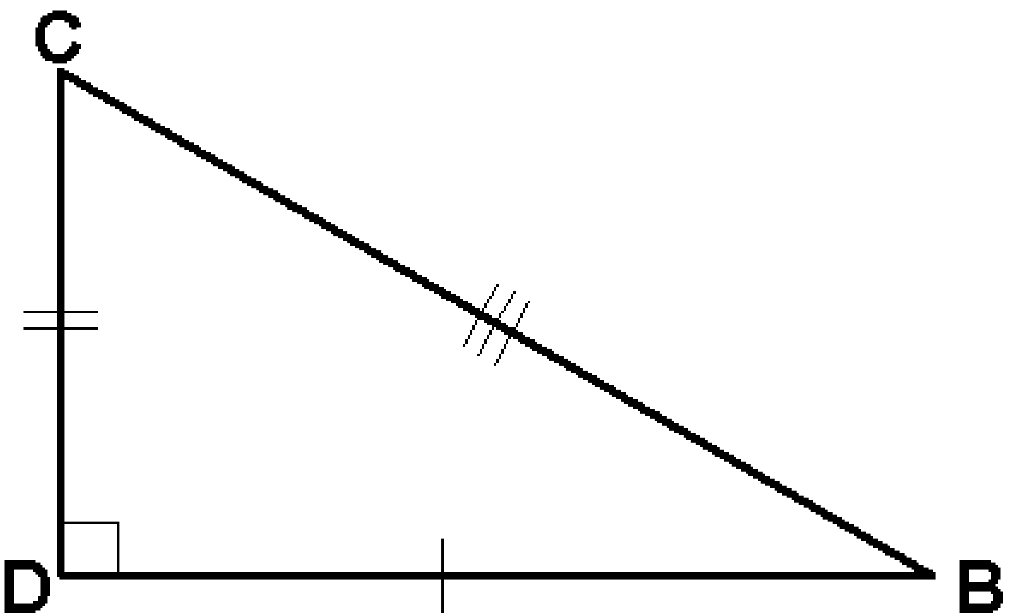
$$\angle\gamma = \angle\beta$$

Our picture becomes. . .



Notice we can dissect this right triangle. We must rotate the first right triangle ¼ turn clockwise so the two triangles have the same alignment.





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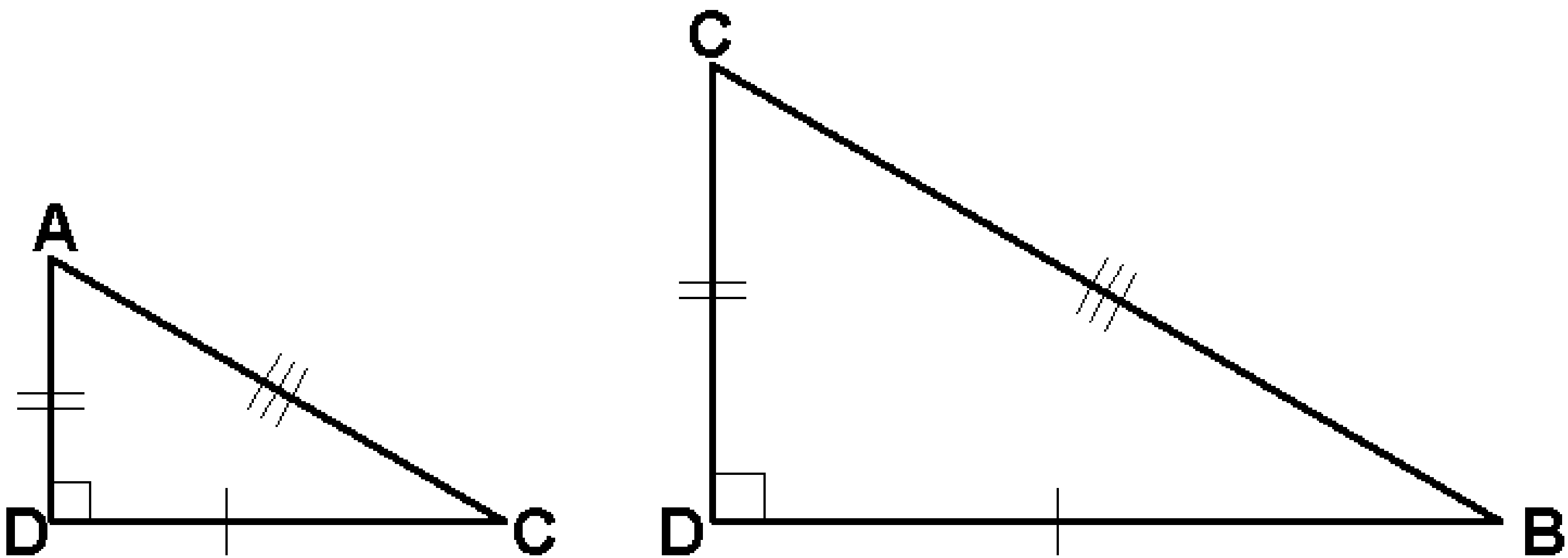
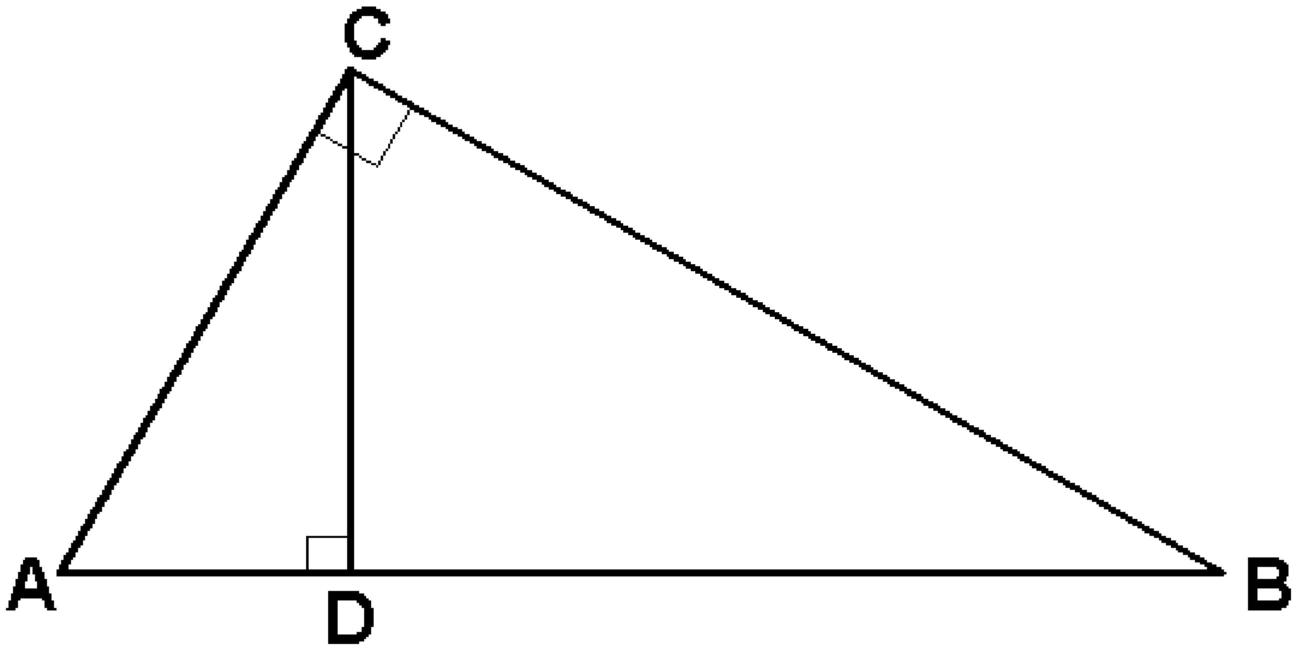
Since these triangles are similar, the following properties can be used.

$$\frac{\overline{AD}}{\overline{AC}} = \frac{\overline{CD}}{\overline{CB}}; \quad \frac{\overline{AD}}{\overline{DC}} = \frac{\overline{CD}}{DB}; \quad \frac{\overline{DC}}{\overline{AC}} = \frac{\overline{DB}}{\overline{CB}}$$



It can be shown that the original right triangle ABC is similar to the smaller two right triangles.



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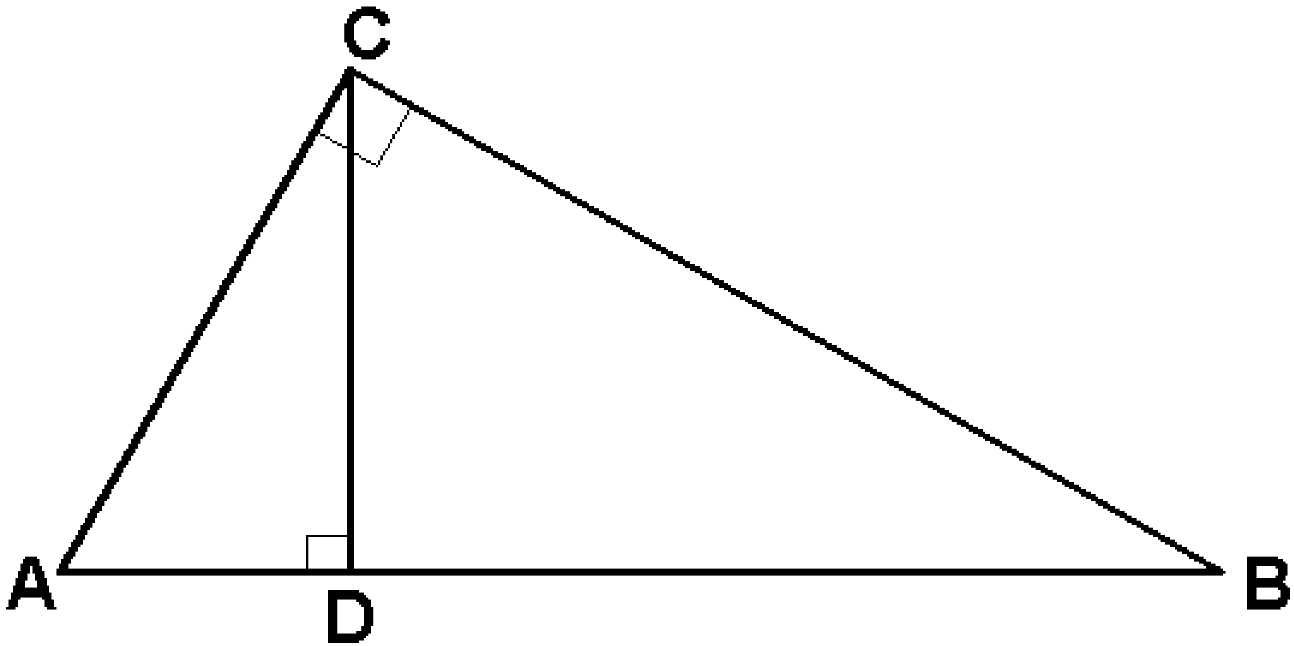
If we separate the figure into three triangles and use the same alignment for all three we get . . .



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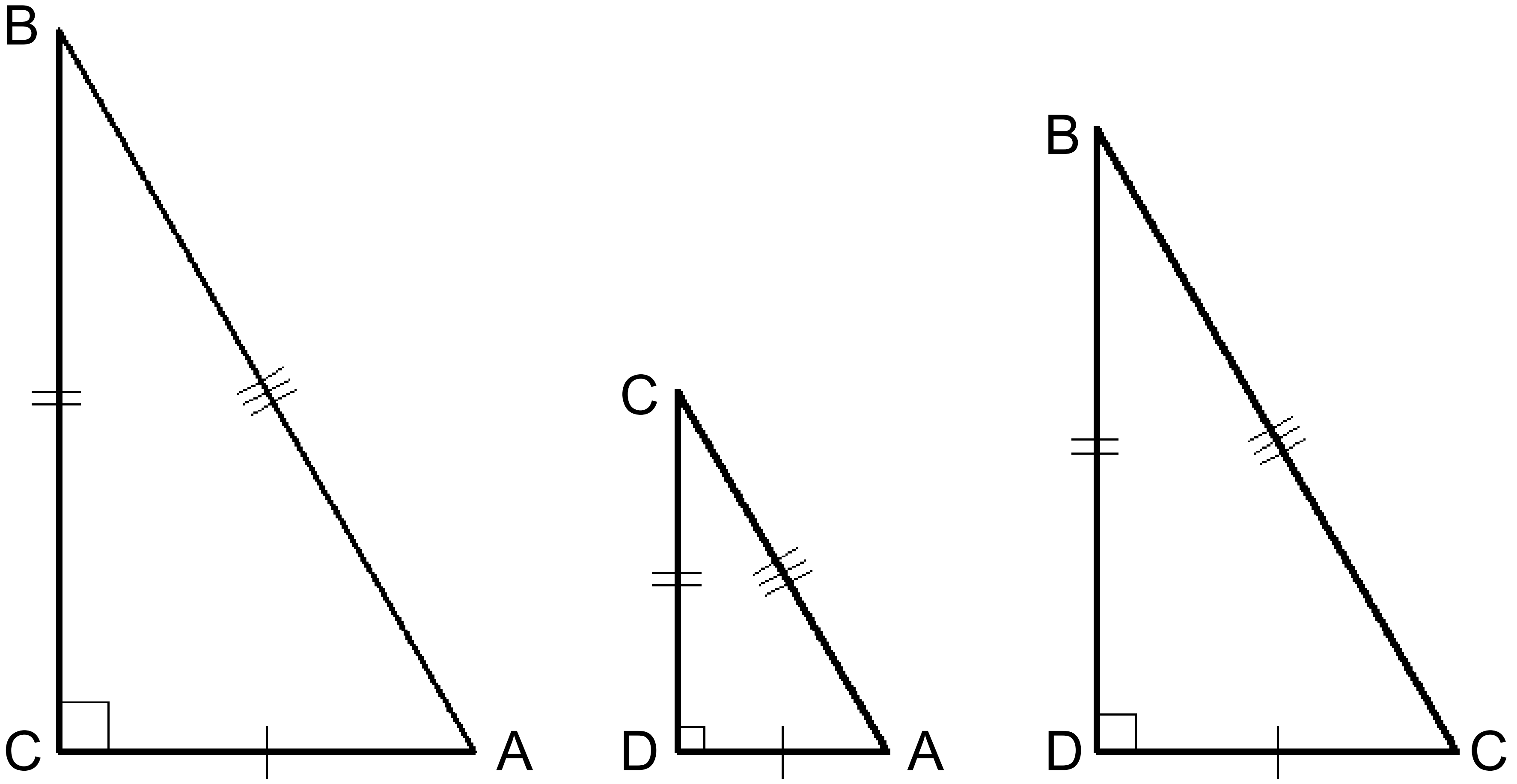
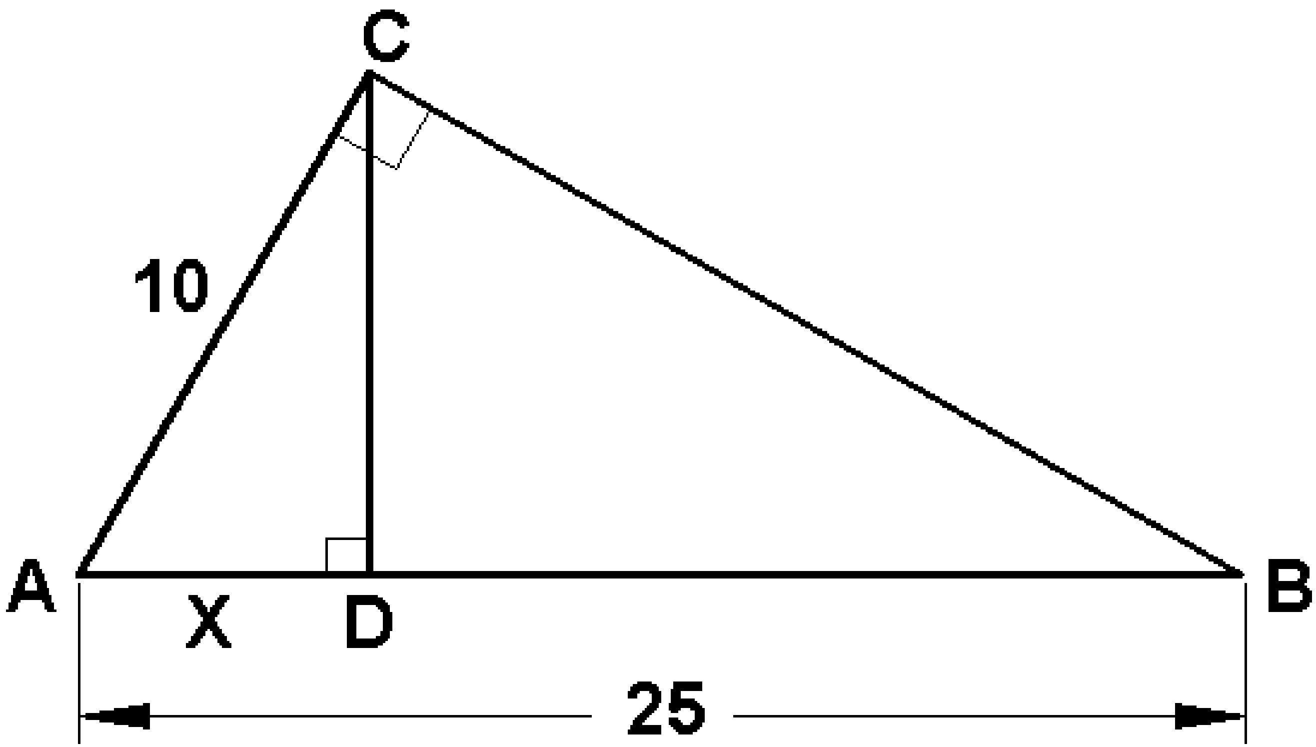


Similar proportions can be created.

Example 1) Determine the value for X



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Since $\dfrac{\overline{AD}}{\overline{AC}} = \dfrac{\overline{AC}}{\overline{AB}}$ $\leftarrow$ short leg $\leftarrow$ hypotenuse

we have $\dfrac{X}{10} = \dfrac{10}{25}$ solving for X,

$$X = 10 \bullet \frac{10}{25}$$

$$X = \frac{100}{25}$$

$$\boxed{X = 4}$$

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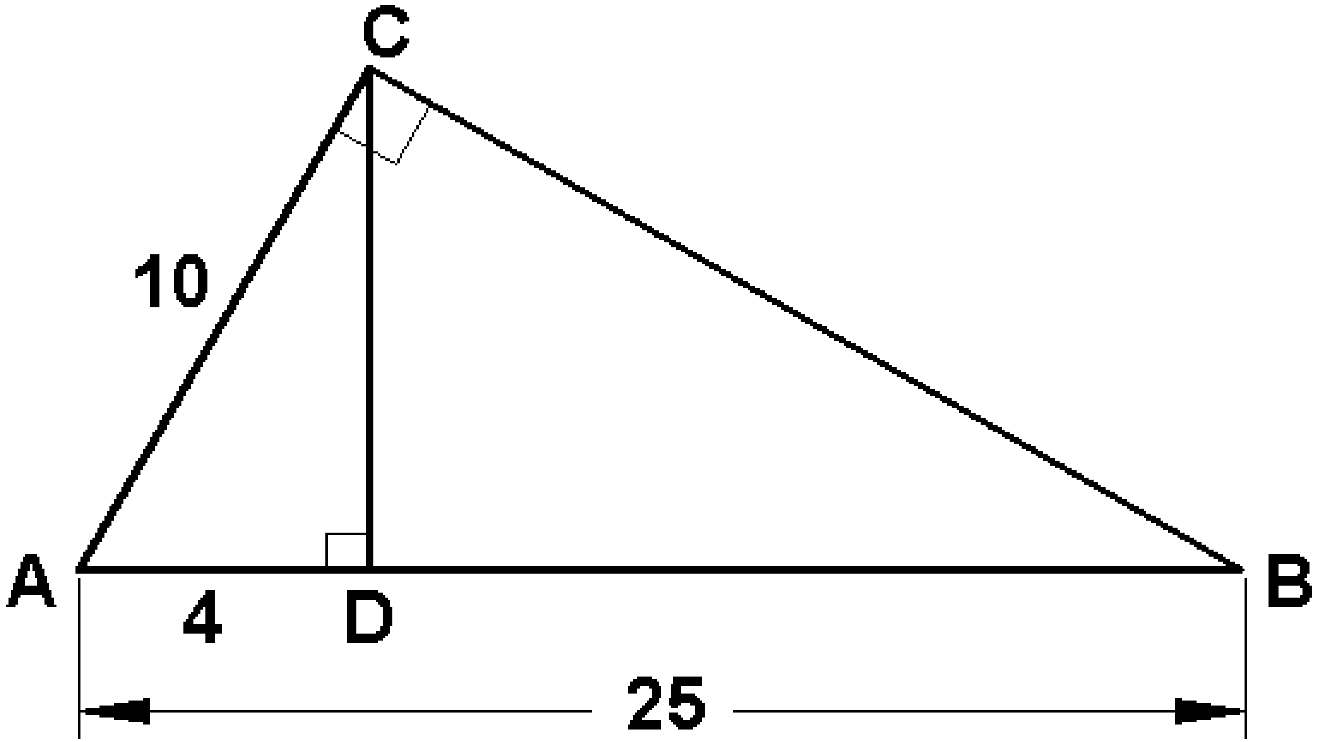
We really have



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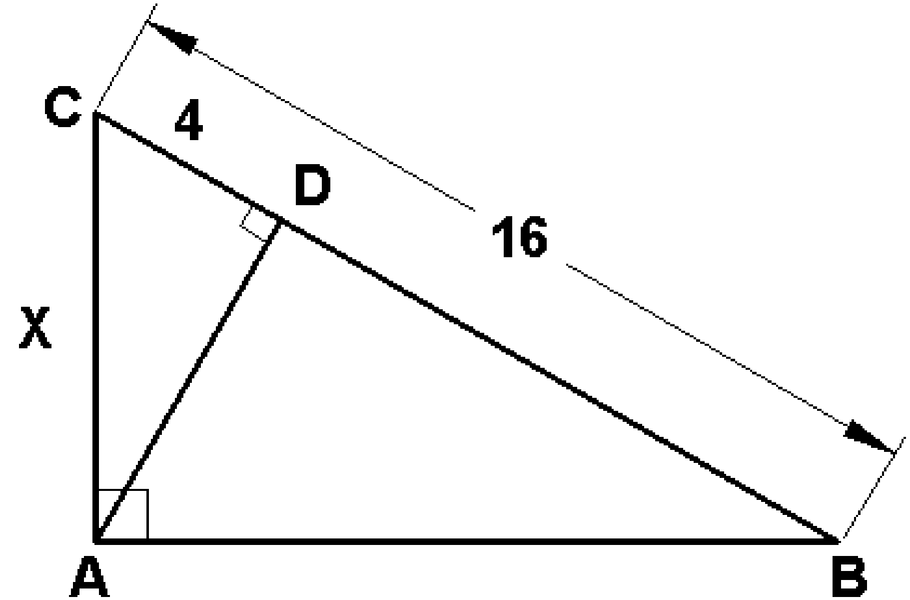
Example 2)



$$\frac{\overline{AC}}{\overline{CB}} = \frac{\overline{CD}}{\overline{CA}}$$ ← short leg ← hypotenuse

becomes $\frac{X}{16} = \frac{4}{X}$; $X^2 = 16(4)$; $X^2 = 64$

$X = \pm\sqrt{64}$; $X = \pm 8$

$\boxed{X = 8}$

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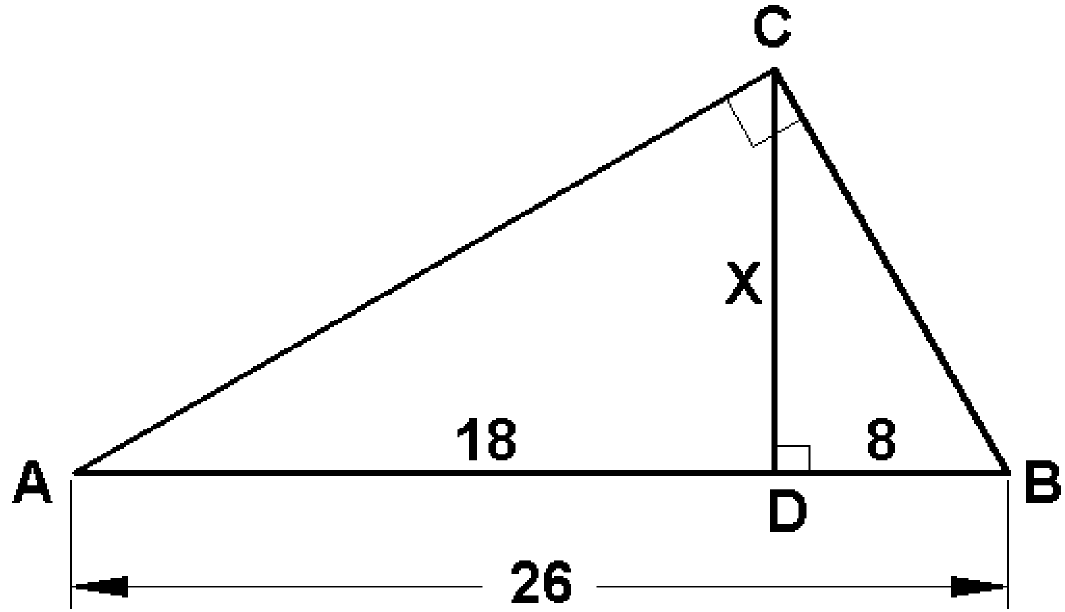
Example 3)

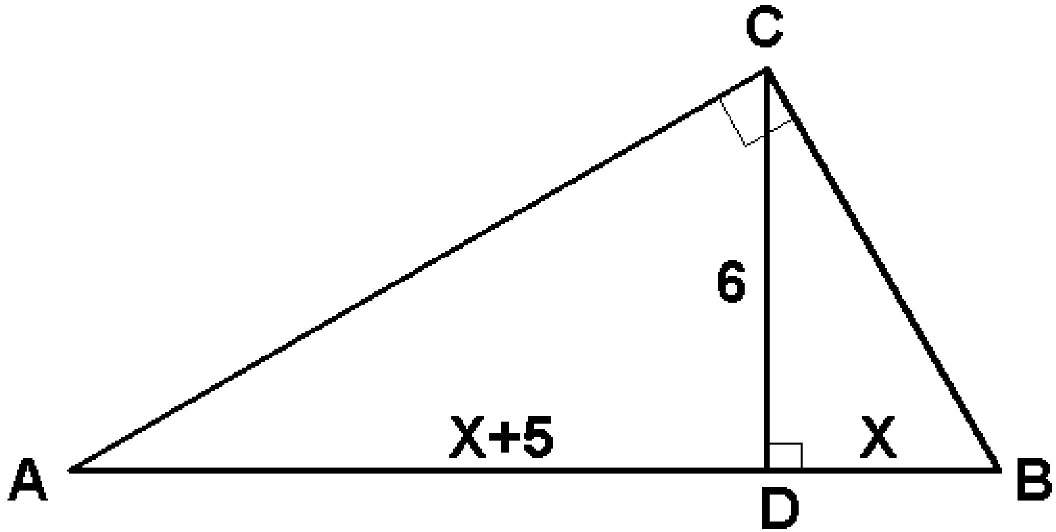


$$\frac{X}{18} = \frac{8}{X}$$ ← short leg
← long leg

$$X^2 = 8(18); \quad X^2 = 144; \quad X = \pm\sqrt{144}$$

$$X = \pm 12; \quad \boxed{X = 12}$$

Example 4)



$$\frac{6}{X+5} = \frac{X}{6}$$ ← short leg ← long leg

$$36 = X(X+5);\ \ 36 = X^2 + 5X;\ \ X^2 + 5X - 36 = 0$$

$$(X+9)(X-4) = 0$$

$X + 9 = 0$	$X - 4 = 0$
$X = -9$	$\boxed{X = 4}$

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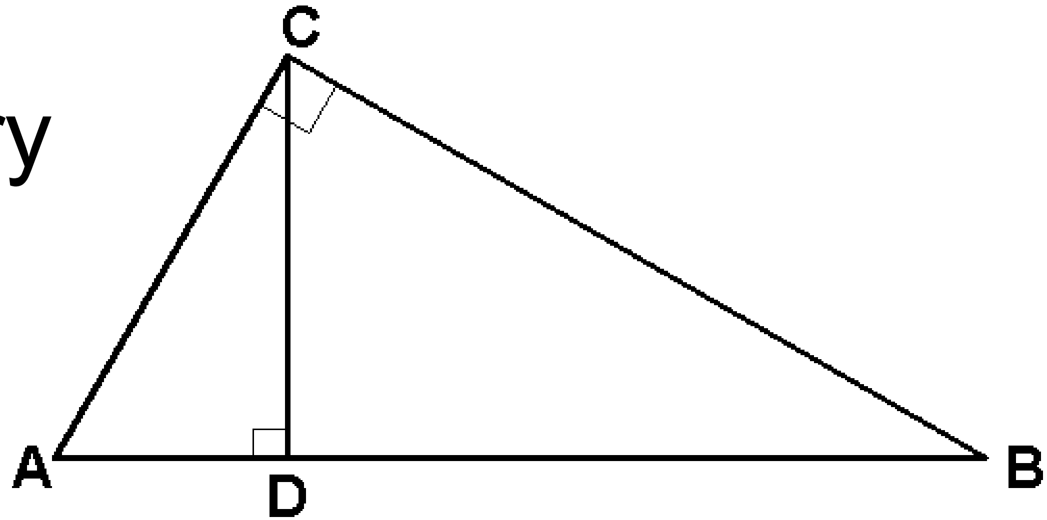
Summary



$$\frac{\overline{AD}}{\overline{AC}} = \frac{\overline{AC}}{\overline{CB}} \qquad \frac{\text{Short Leg}}{\text{Hypotenuse}}$$

$$\frac{\overline{DB}}{\overline{CB}} = \frac{\overline{CB}}{\overline{AB}} \qquad \frac{\text{Long Leg}}{\text{Hypotenuse}}$$

$$\frac{\overline{AD}}{\overline{CD}} = \frac{\overline{CD}}{\overline{DB}} \qquad \frac{\text{Short Leg}}{\text{Long leg}}$$

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End of Right Triangles

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