Law of Sines
Solving Oblique Triangles

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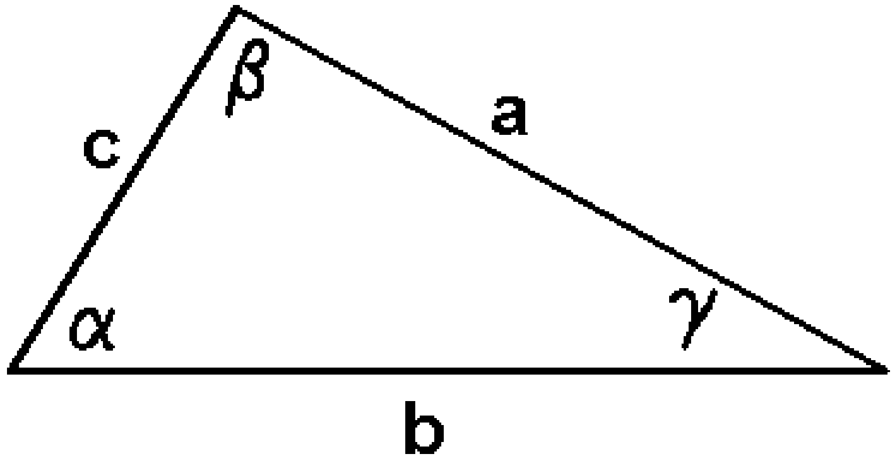
TOPICS BACK NEXT EXIT

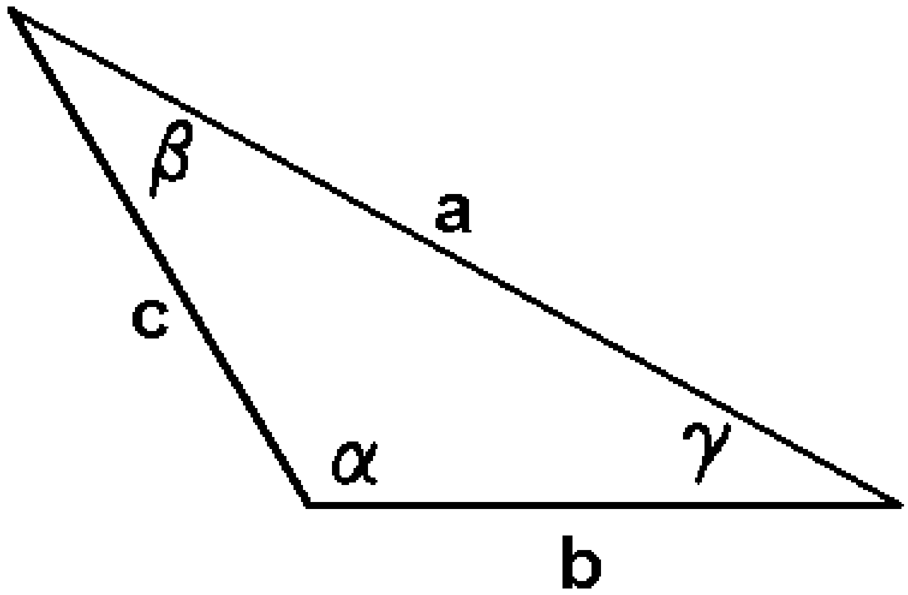
Trigonometry can help us solve non-right triangles as well. Non-right triangles are know as oblique triangles. There are two categories of oblique triangles—acute and obtuse.

Acute Triangles



In an acute triangle, each of the angles is less than 90º.

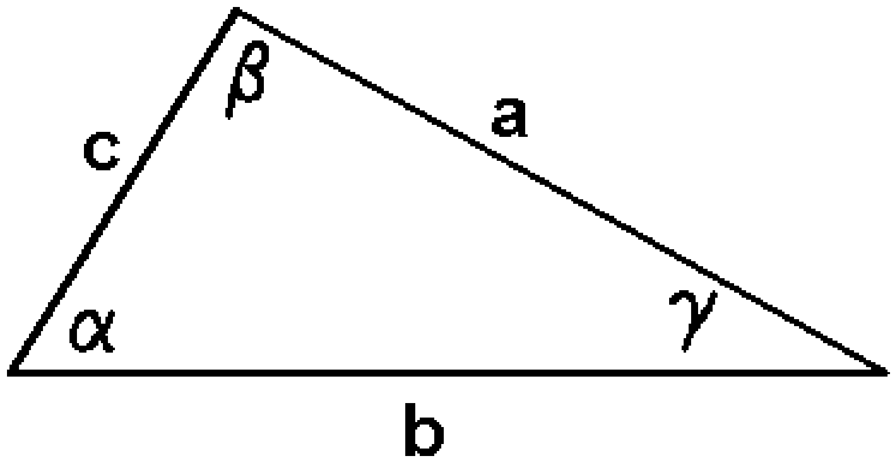
Obtuse Triangles



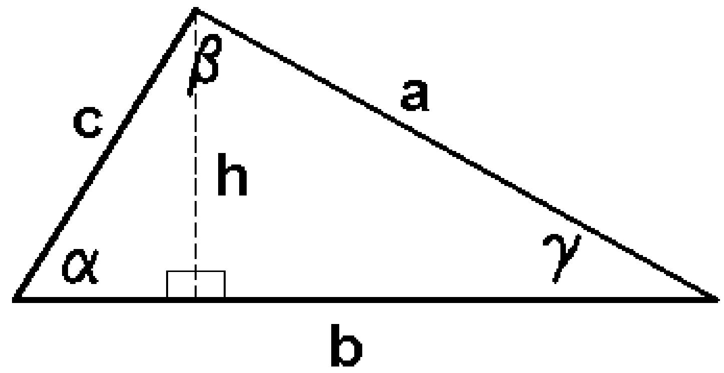
In an obtuse triangle, one of the angles is obtuse *(between 90º and 180º)*. Can there be two obtuse angles in a triangle?

The Law of Sines

Consider the first category, an acute triangle (α, β, γ are acute).



Create an altitude, h.



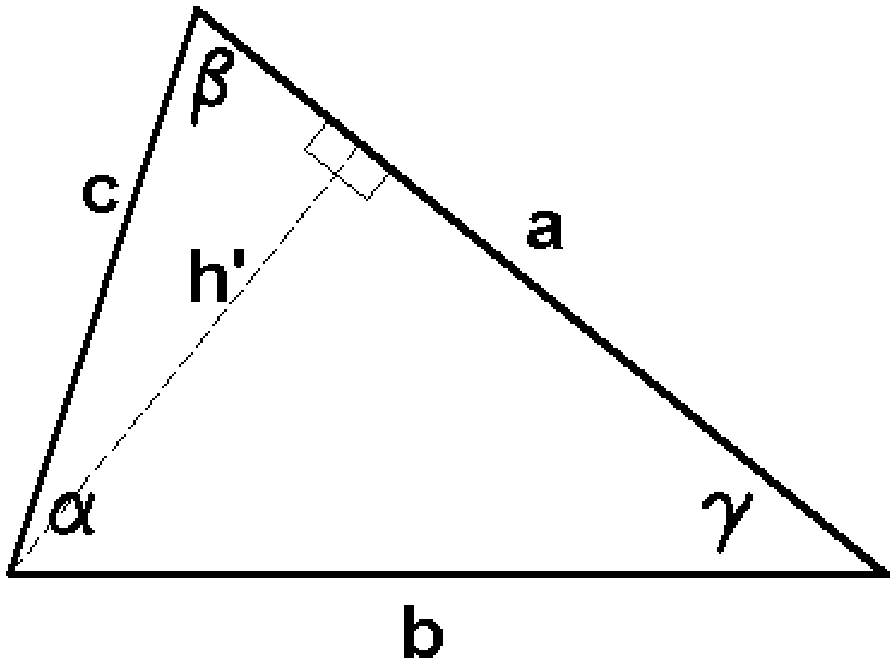
Now, $\sin(\gamma) = \dfrac{h}{a}$, so that $h = a \bullet \sin(\gamma)$

But $\sin(\alpha) = \dfrac{h}{c}$, so that $h = c \bullet \sin(\alpha)$

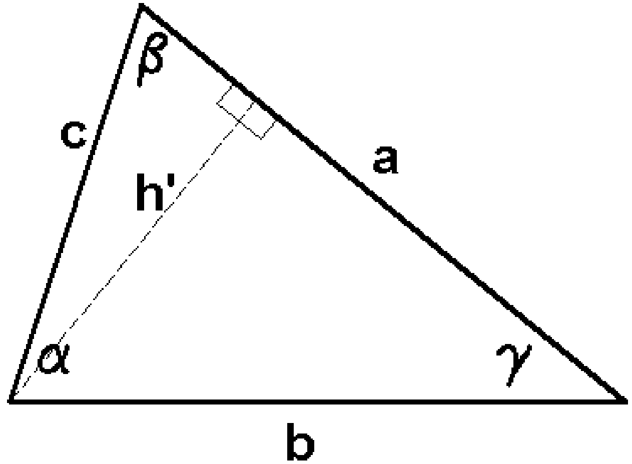
By transitivity, $a \bullet \sin(\gamma) = c \bullet \sin(\alpha)$

Which means $\dfrac{\sin(\gamma)}{c} = \dfrac{\sin(\alpha)}{a}$

Let’s create another altitude h’.



TOPICS BACK NEXT EXIT



$$\sin(\beta)=\frac{h'}{c},\quad \text{so that}\quad h'= c\bullet\sin(\beta)$$

$$\sin(\gamma)=\frac{h'}{b},\quad \text{so that}\quad h'= b\bullet\sin(\gamma)$$

By transitivity, $c\bullet\sin(\beta)= b\bullet\sin(\gamma)$

Which means $\dfrac{\sin(\beta)}{b}=\dfrac{\sin(\gamma)}{c}$

TOPICS BACK NEXT EXIT

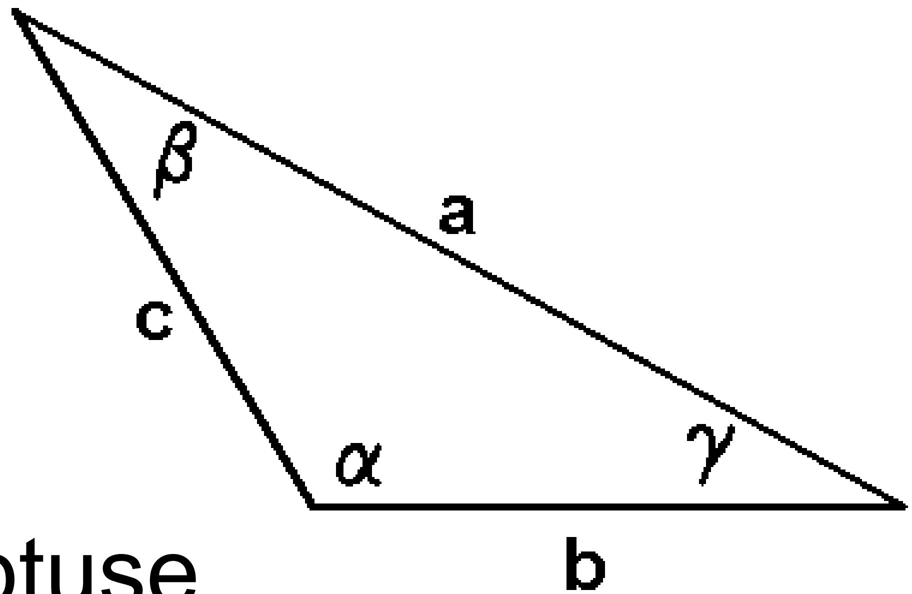
Putting these together, we get

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

This is known as the Law of Sines.

The Law of Sines is used when we know any two angles and one side or when we know two sides and an angle opposite one of those sides.

Fact The law of sines also works for oblique triangles that contain an obtuse angle (angle between 90º and 180º).



α is obtuse

TOPICS BACK NEXT EXIT

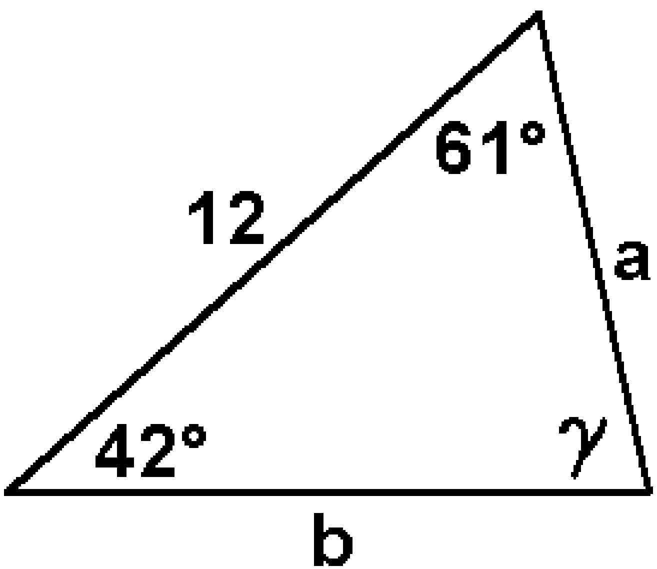
General Strategies for Using the Law of Sines

One side and two angles are known.

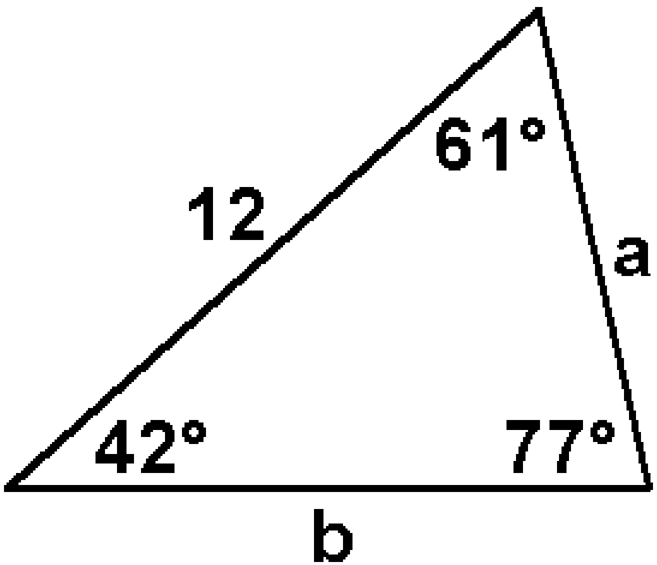
ASA or SAA

ASA

From the model, we need to determine a, b, and γ using the law of sines.

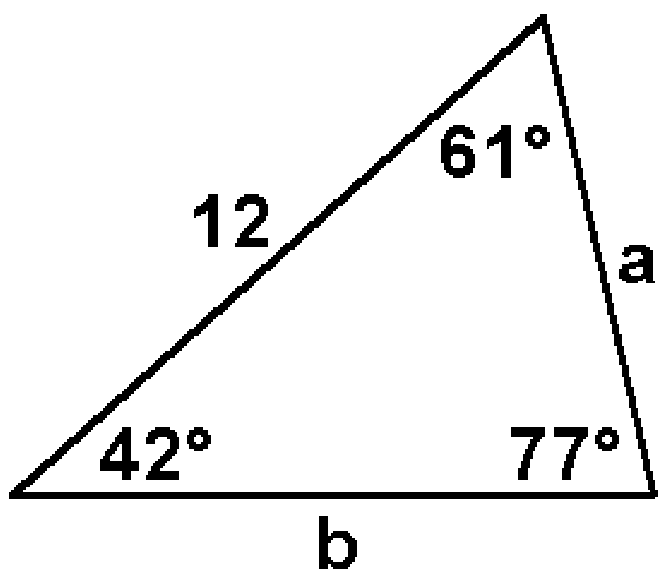


First off, $42^\circ + 61^\circ + \gamma = 180^\circ$ so that $\gamma = 77^\circ$. (Knowledge of two angles yields the third!)



Now by the law of sines, we have the following relationships:

$$\frac{\sin(42^\circ)}{a} = \frac{\sin(77^\circ)}{12} \quad ; \quad \frac{\sin(61^\circ)}{b} = \frac{\sin(77^\circ)}{12}$$



So that

$$a = \frac{12 \bullet \sin(42^\circ)}{\sin(77^\circ)}$$

$$a \approx \frac{12 \bullet (0.6691)}{0.9744}$$

$$\boxed{a \approx 8.2401}$$

$$b = \frac{12 \bullet \sin(61^\circ)}{\sin(77^\circ)}$$

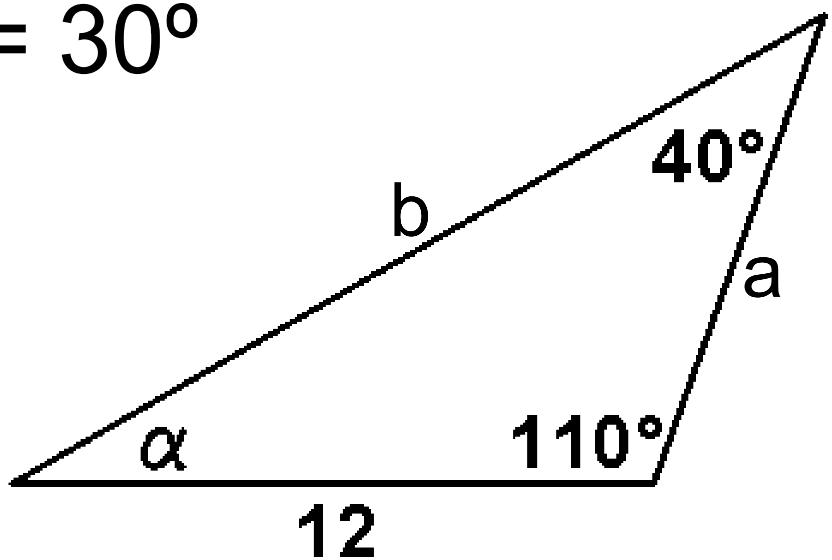
$$b \approx \frac{12 \bullet (0.8746)}{0.9744}$$

$$\boxed{b \approx 10.7709}$$

SAA

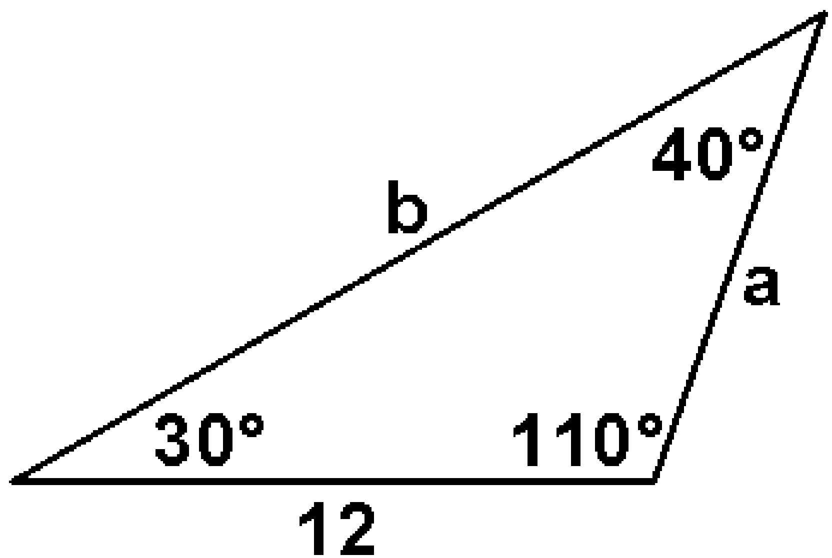
From the model, we need to determine a, b, and α using the law of sines.

Note: $\alpha + 110° + 40° = 180°$
so that $\alpha = 30°$



By the law of sines,

$$\frac{\sin(30^\circ)}{a} = \frac{\sin(40^\circ)}{12} \quad ; \quad \frac{\sin(110^\circ)}{b} = \frac{\sin(40^\circ)}{12}$$



TOPICS BACK NEXT EXIT

Thus,

$$a = \frac{12 \cdot \sin(30^\circ)}{\sin(40^\circ)}$$

$$a \approx \frac{12 \cdot (0.5)}{0.6428}$$

$$\boxed{a \approx 9.3341}$$

$$b = \frac{12 \cdot \sin(110^\circ)}{\sin(40^\circ)}$$

$$b \approx \frac{12 \cdot (0.9397)}{0.5}$$

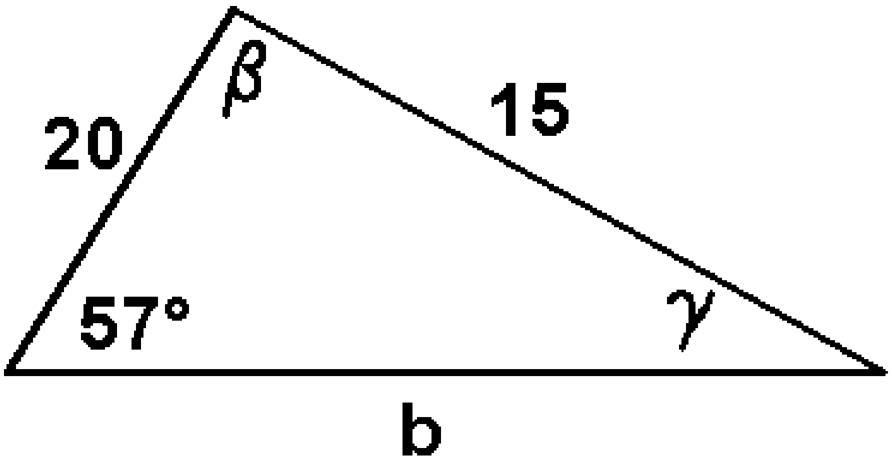
$$\boxed{b \approx 22.5526}$$

The Ambiguous Case – SSA

In this case, you may have information that results in one triangle, two triangles, or no triangles.

SSA – No Solution

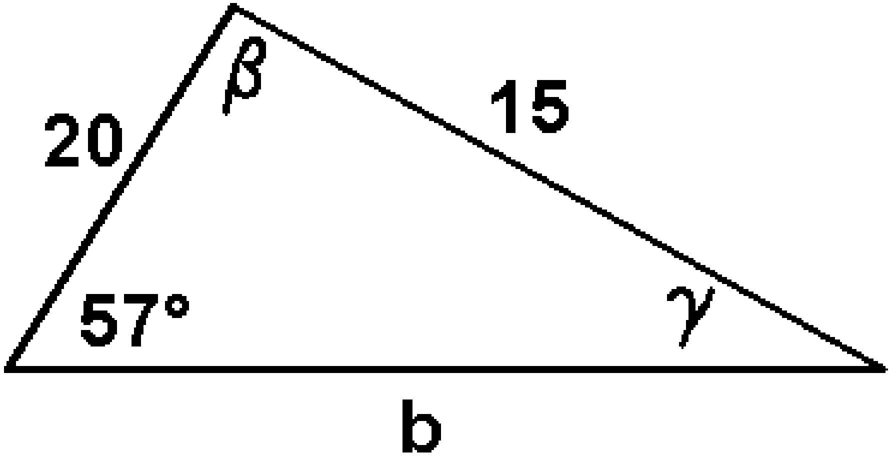
Two sides and an angle opposite one of the sides.



TOPICS BACK NEXT EXIT

By the law of sines,

$$\frac{\sin(57^\circ)}{15} = \frac{\sin(\gamma)}{20}$$



TOPICS BACK NEXT EXIT

Thus,

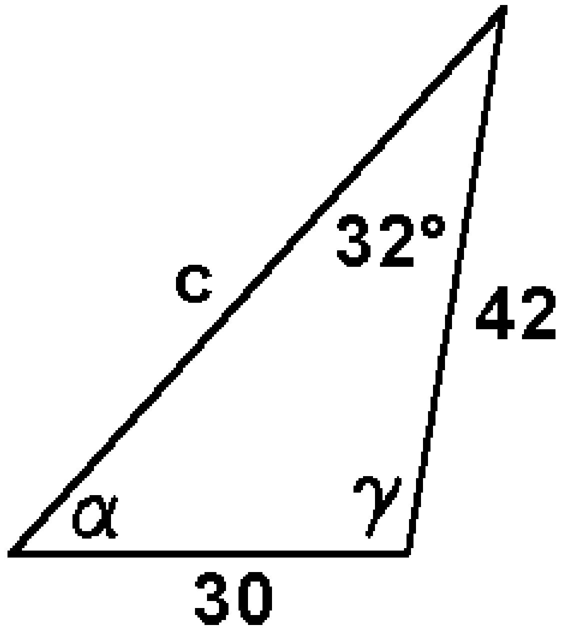
$$\sin(\gamma) = \frac{20 \bullet \sin(57^\circ)}{15}$$

$$\sin(\gamma) = \frac{20 \bullet (0.8387)}{15}$$

$$\sin(\gamma) = 1.1183 \leftarrow \text{ Impossible!}$$

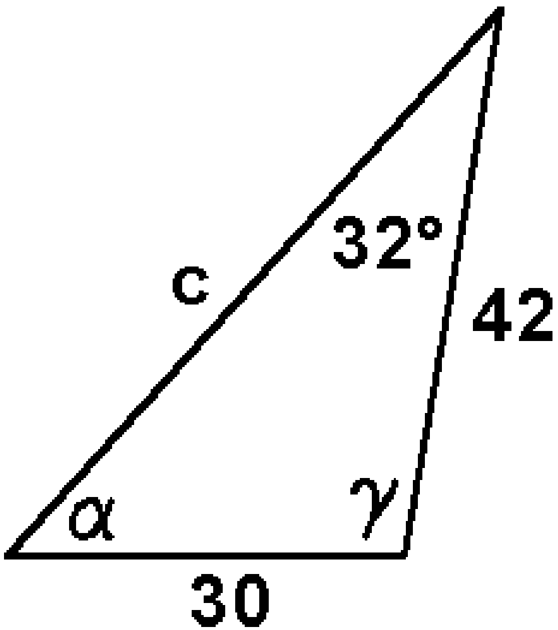
Therefore, there is no value for γ that exists! No Solution!

SSA – Two Solutions



By the law of sines,

$$\frac{\sin(32^\circ)}{30} = \frac{\sin(\alpha)}{42}$$



So that,

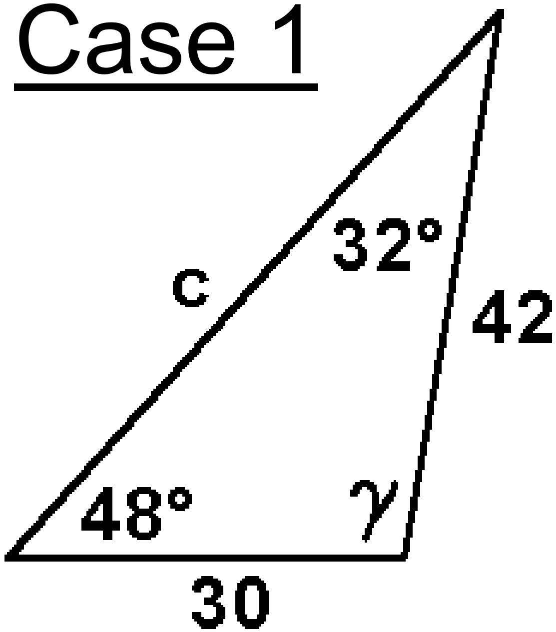
$$\sin(\alpha) = \frac{42 \bullet \sin(32^\circ)}{30}$$

$$\sin(\alpha) = \frac{42 \bullet (0.5299)}{30}$$

$$\sin(\alpha) = 0.7419$$

$$\alpha = 48^\circ \text{ or } 132^\circ$$

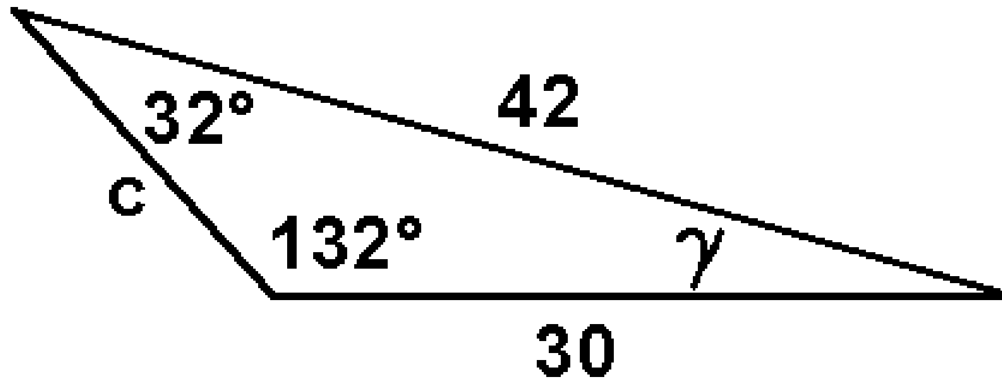
Case 1



$$48^\circ + 32^\circ + \gamma = 180^\circ$$

$$\gamma = 100^\circ$$

Case 2

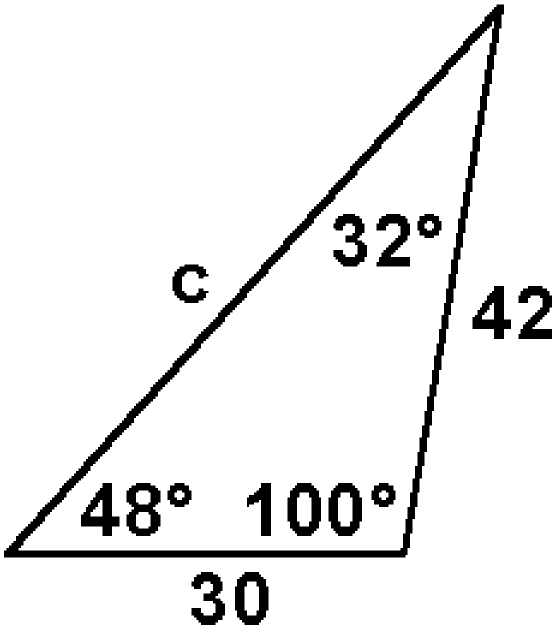


$$132^\circ + 32^\circ + \gamma = 180^\circ$$

$$\gamma = 16^\circ$$

Both triangles are valid! Therefore, we have two solutions.

Case 1



$$\frac{\sin(100^\circ)}{c} = \frac{\sin(32^\circ)}{30}$$

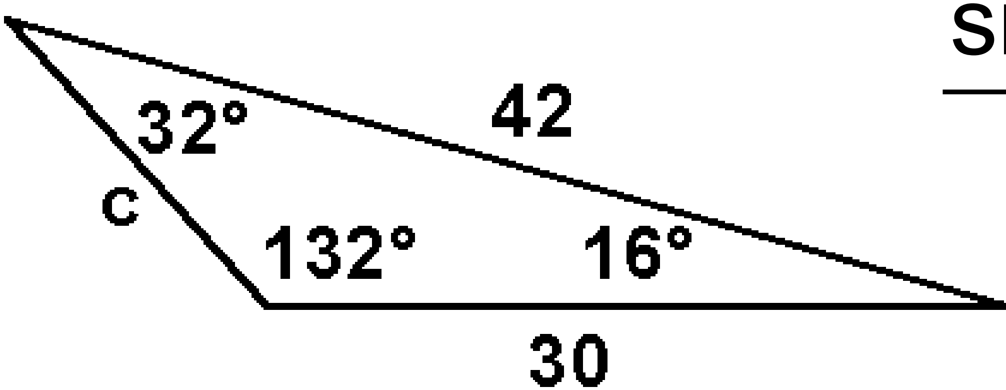
$$c = \frac{30 \bullet \sin(100^\circ)}{\sin(32^\circ)}$$

$$c \approx \frac{30 \bullet (0.9848)}{0.5299}$$

$$\boxed{c \approx 55.7539}$$

TOPICS BACK NEXT EXIT

Case 2



$$\frac{\sin(16^\circ)}{c} = \frac{\sin(32^\circ)}{30}$$

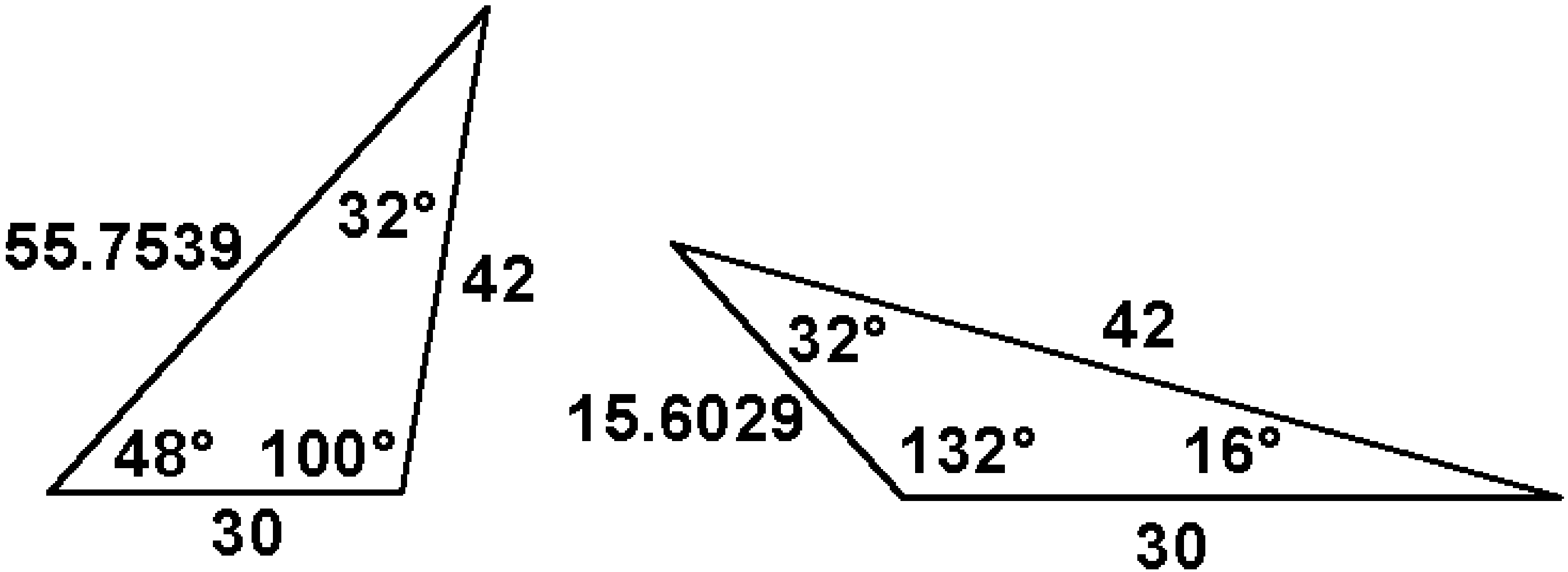
$$c = \frac{30 \bullet \sin(16^\circ)}{\sin(32^\circ)}$$

$$c \approx \frac{30 \bullet (0.2756)}{0.5299}$$

$$\boxed{c \approx 15.6029}$$

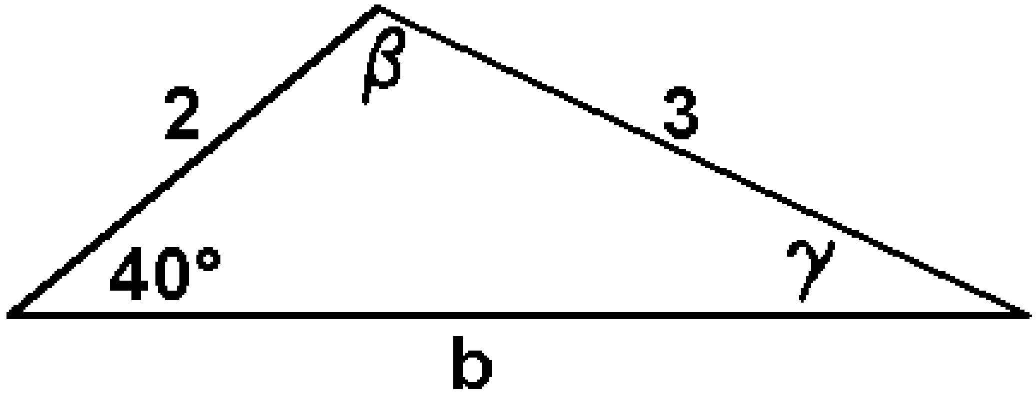
TOPICS BACK NEXT EXIT

Finally our two solutions:



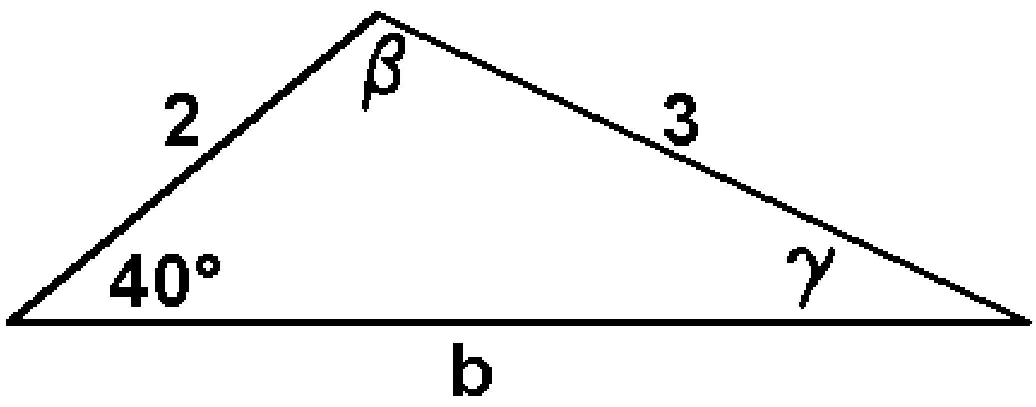
TOPICS BACK NEXT EXIT

SSA – One Solution



By the law of sines,

$$\frac{\sin(40^\circ)}{3} = \frac{\sin(\gamma)}{2}$$



TOPICS BACK NEXT EXIT

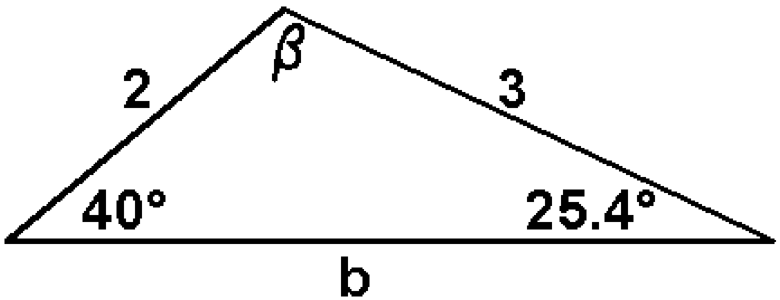
$$\sin(\gamma) = \frac{2 \bullet \sin(40^\circ)}{3}$$

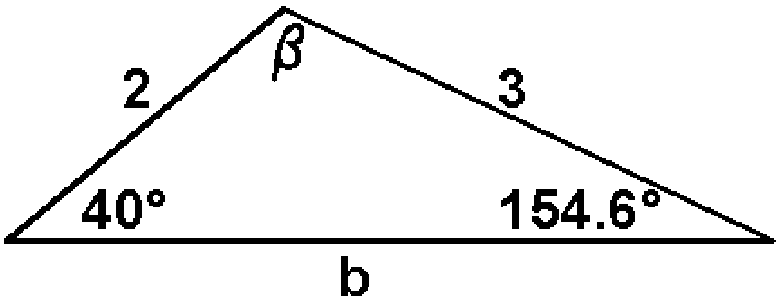
$$\sin(\gamma) = \frac{2 \bullet (0.6428)}{3}$$

$$\sin(\gamma) = 0.4285$$

$$\gamma = 25.4^\circ \text{ or } 154.6^\circ$$

Note– Only one is legitimate!





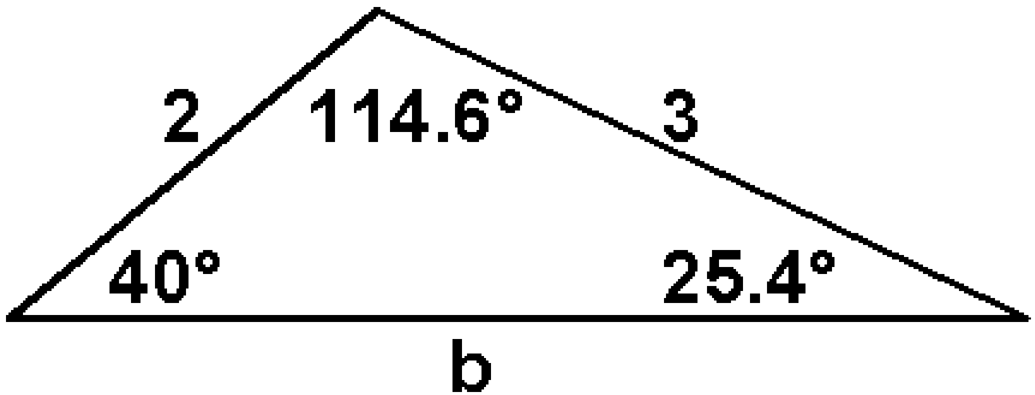
$$40^\circ + \beta + 25.4^\circ = 180^\circ$$

$$\beta = 114.6^\circ$$

$$40^\circ + \beta + 154.6^\circ = 180^\circ$$

$$\beta = -14.6^\circ \leftarrow \text{Not Possible!}$$

Thus we have only one triangle.



By the law of sines,

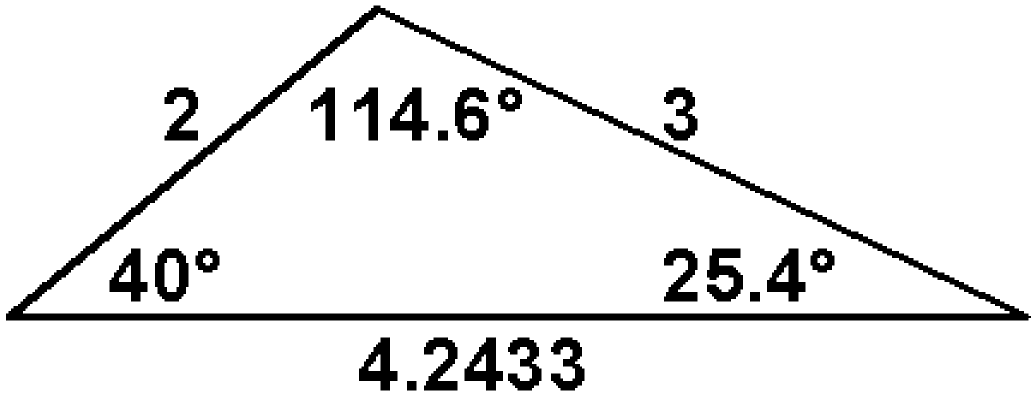
$$\frac{\sin(114.6^\circ)}{b} = \frac{\sin(40^\circ)}{3};$$

$$b = \frac{3 \bullet \sin(114.6^\circ)}{\sin(40^\circ)}$$

$$b \approx \frac{3 \bullet (0.9092)}{0.6428}$$

$$\boxed{b \approx 4.2433}$$

Finally, we have:



TOPICS BACK NEXT EXIT

End of Law of Sines

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