

The Law of Cosines

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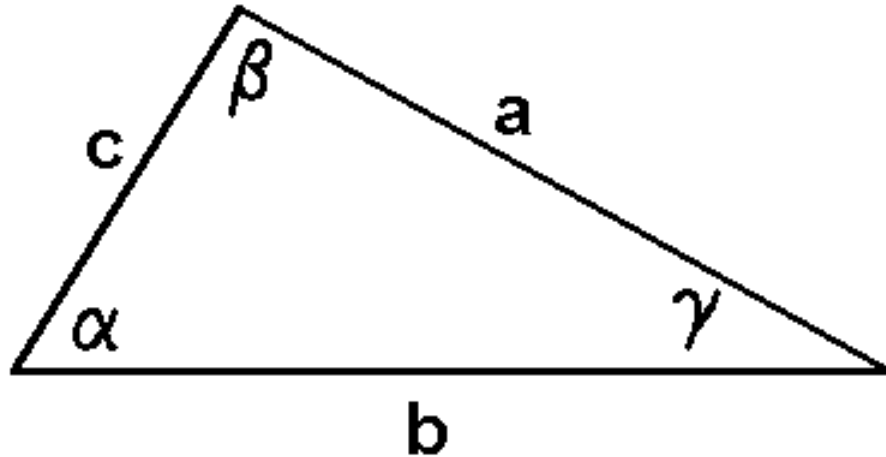
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When solving an oblique triangle, using one of three available equations utilizing the cosine of an angle is handy. The equations are as follows:

$$1) a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

$$2) b^2 = a^2 + c^2 - 2ac \cos(\beta)$$

$$3) c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$

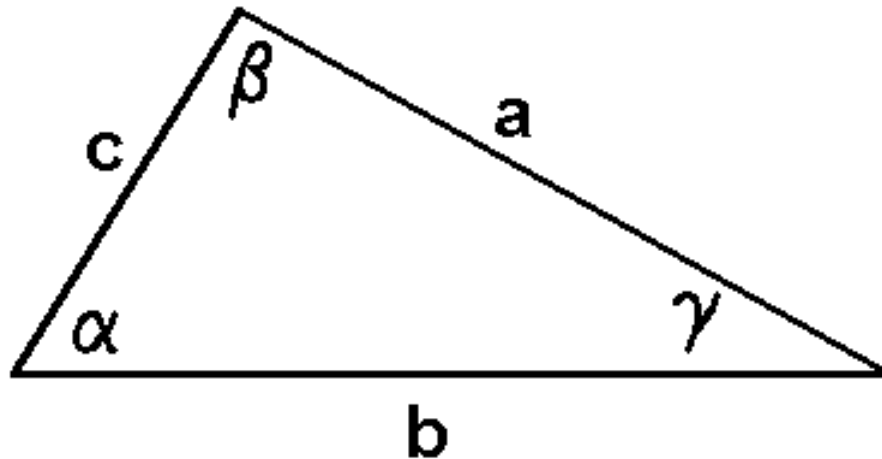


Note:

The angle opposite a in equation 1 is α .

The angle opposite b in equation 2 is β .

The angle opposite c in equation 3 is γ .



Where did these three equations come from?

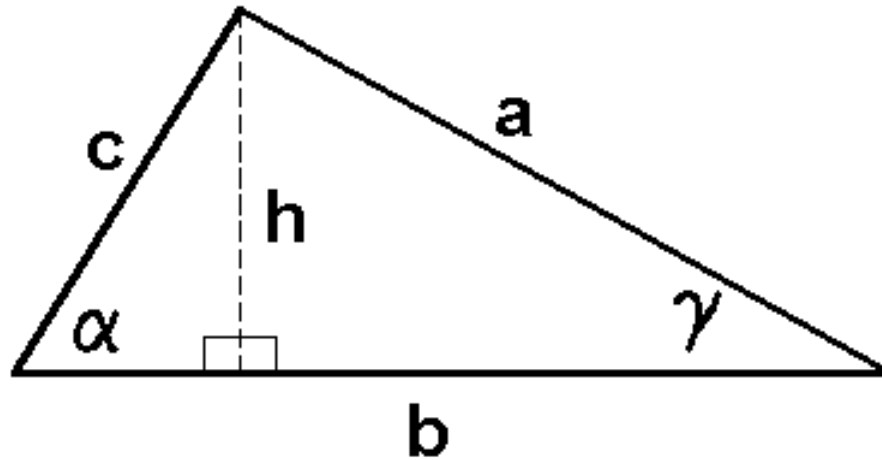
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Create an altitude h .



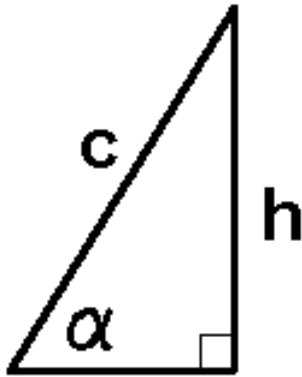
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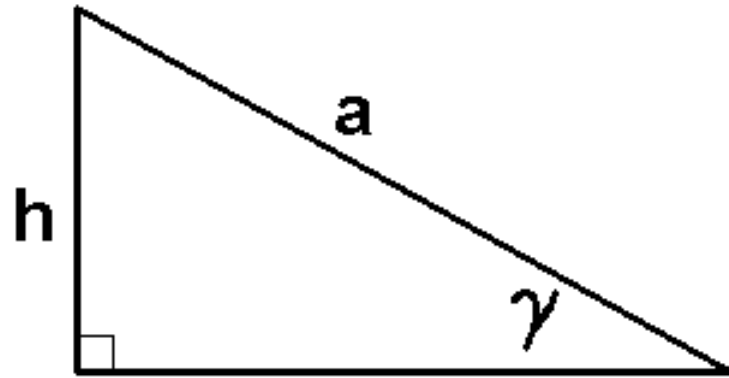
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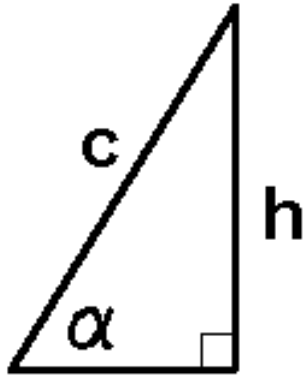
We've split our original oblique triangle into two triangles.



First Triangle



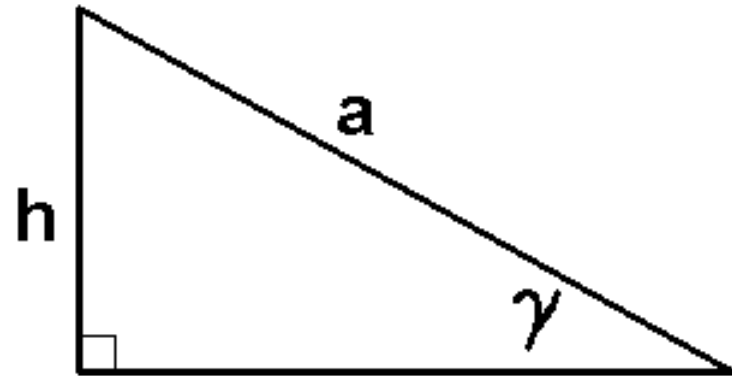
Second Triangle



First Triangle

$$\sin(\alpha) = \frac{h}{c}$$

$$\text{Thus } h = c \cdot \sin(\alpha)$$

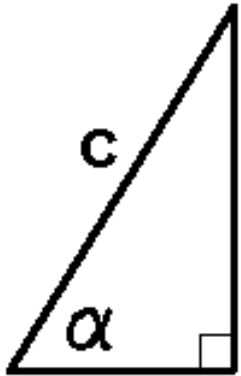


Second Triangle

$$\sin(\gamma) = \frac{h}{a}$$

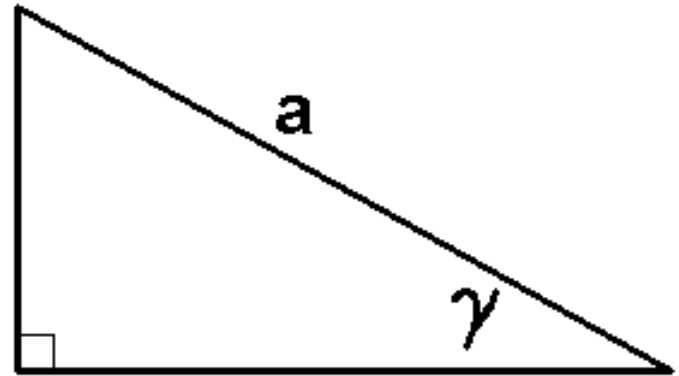
$$\text{Thus } h = a \cdot \sin(\gamma)$$

Our picture becomes:



$$c \cdot \sin(\alpha)$$

$$a \cdot \sin(\gamma)$$



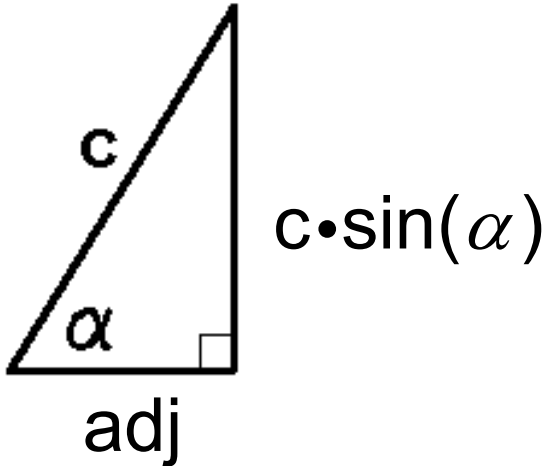
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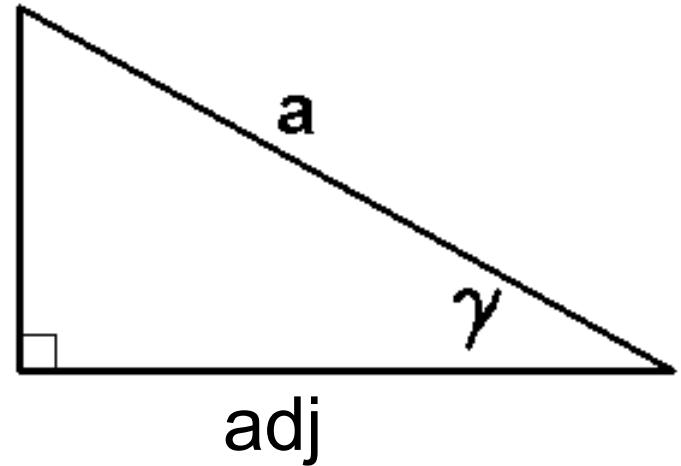
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Note the base of our triangles.



$$a \cdot \sin(\gamma)$$



First Triangle

$$\cos(\alpha) = \frac{\text{adj}}{c}$$

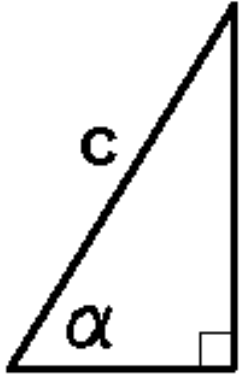
$$\text{adj} = c \cdot \cos(\alpha)$$

Second Triangle

$$\cos(\gamma) = \frac{\text{adj}}{a}$$

$$\text{adj} = a \cdot \cos(\gamma)$$

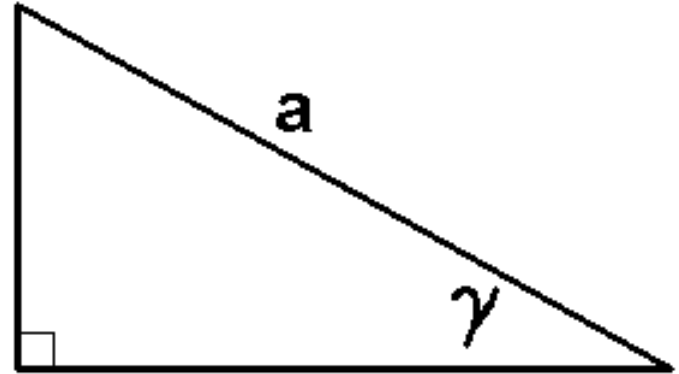
Our triangles now become,



$$c \cdot \sin(\alpha)$$

$$c \cdot \cos(\alpha)$$

$$a \cdot \sin(\gamma)$$



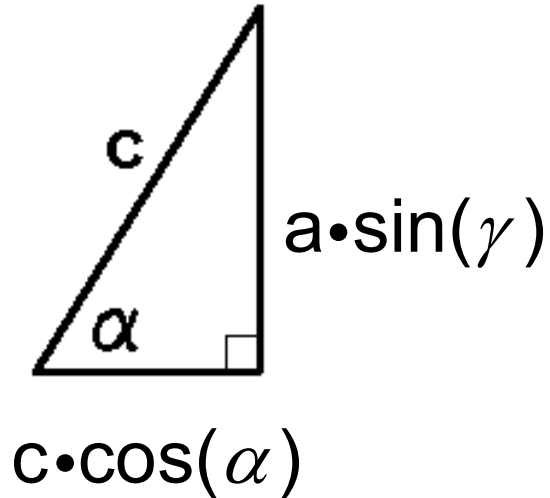
$$a \cdot \cos(\gamma)$$

*Consider two important relationships:

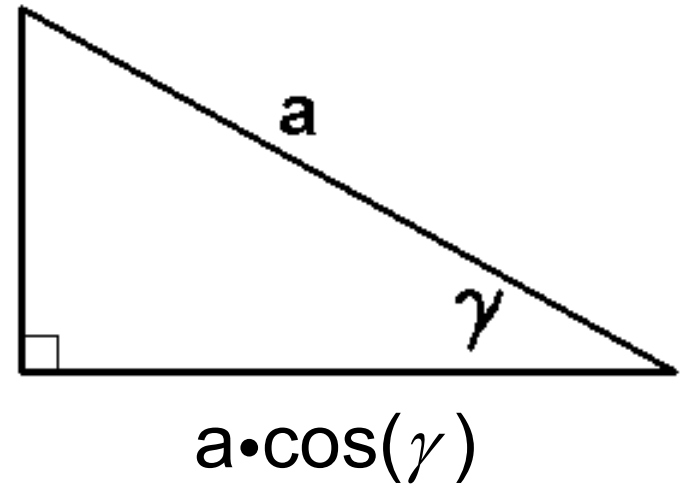
$$1) \quad c \cdot \sin(\alpha) = a \cdot \sin(\gamma)$$

$$2) \quad c \cdot \cos(\alpha) + a \cdot \cos(\gamma) = b$$

Using Relationship 1, we obtain:

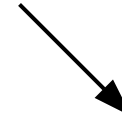
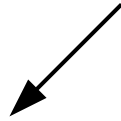


$$c \cdot \sin(\alpha)$$



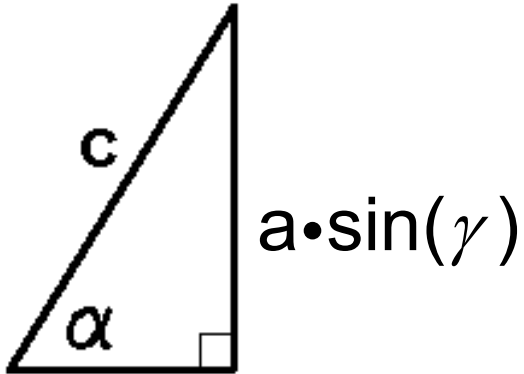
Take a closer look at Relationship 2.

$$c \cdot \cos(\alpha) + a \cdot \cos(\gamma) = b$$



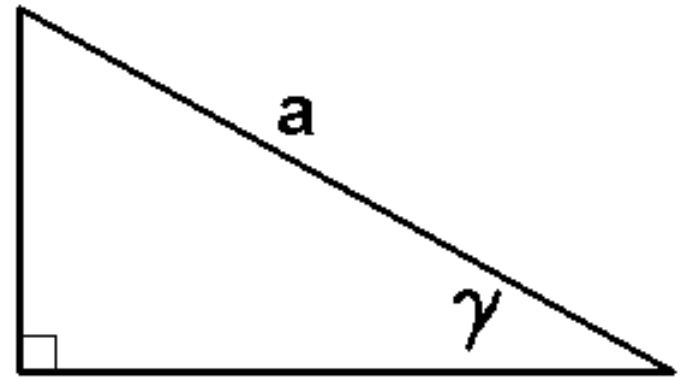
$$c \cdot \cos(\alpha) = b - a \cdot \cos(\gamma) \quad a \cdot \cos(\gamma) = b - c \cdot \cos(\alpha)$$

We now have,



$$b - a \cdot \cos(\gamma)$$

$$c \cdot \sin(\alpha)$$



$$b - c \cdot \cos(\alpha)$$

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Now, by the Pythagorean Theorem,

First Triangle

$$\begin{aligned}c^2 &= [a \sin(\gamma)]^2 + [b - a \cos(\gamma)]^2 \\&= a^2 \sin^2(\gamma) + b^2 - ab \cos(\gamma) - ab \cos(\gamma) + a^2 \cos^2(\gamma) \\&= a^2 \sin^2(\gamma) + a^2 \cos^2(\gamma) + b^2 - 2ab \cos(\gamma) \\&= a^2 [\sin^2(\gamma) + \cos^2(\gamma)] + b^2 - 2ab \cos(\gamma)\end{aligned}$$

But $\sin^2(\gamma) + \cos^2(\gamma) = 1$

Thus $c^2 = a^2 + b^2 - 2ab \cos(\gamma)$

Second Triangle

$$\begin{aligned}a^2 &= [c \sin(\alpha)]^2 + [b - c \cos(\alpha)]^2 \\&= c^2 \sin^2(\alpha) + b^2 - bc \cos(\alpha) - bc \cos(\alpha) + c^2 \cos^2(\alpha) \\&= c^2 \sin^2(\alpha) + c^2 \cos^2(\alpha) + b^2 - 2bc \cos(\alpha) \\&= c^2 [\sin^2(\alpha) + \cos^2(\alpha)] + b^2 - 2bc \cos(\alpha)\end{aligned}$$

also, $\sin^2(\alpha) + \cos^2(\alpha) = 1$

thus $a^2 = b^2 + c^2 - 2bc \cos(\alpha)$

Why don't you try the third equation.

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General Strategies for Using the Law of Cosines

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The formula for the Law of Cosines makes use of three sides and the angle opposite one of those sides. We can use the Law of Cosines:

a. if we know two sides and the included angle, or

b. if we know all three sides of a triangle.

Two sides and one angles are known.

SAS

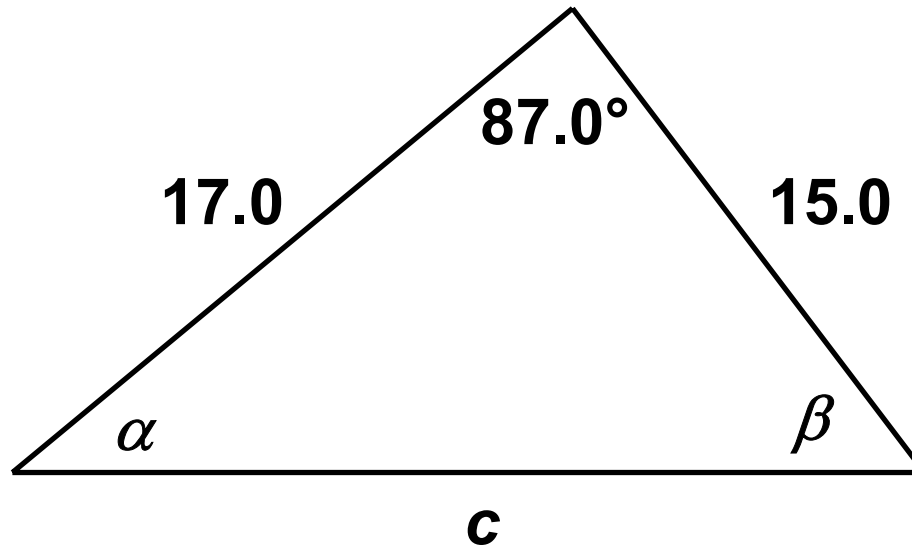
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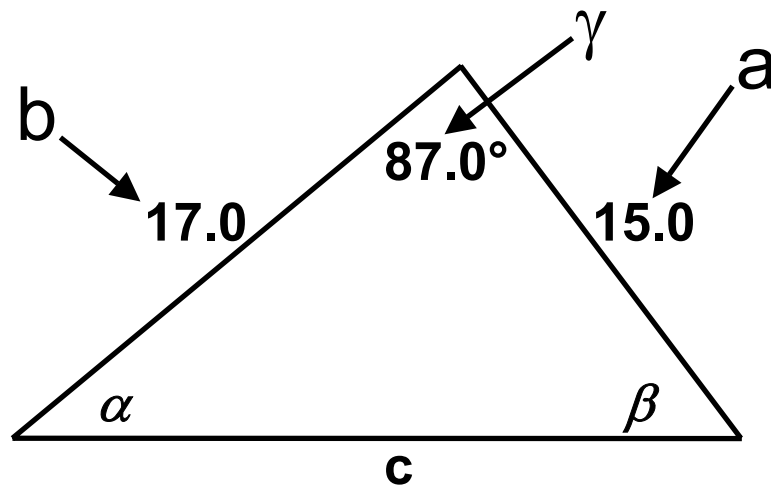


From the model, we need to determine c , α , and β . We start by applying the law of cosines.

To solve for the missing side in this model, we use the form:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

In this form, γ is the angle between a and b , and c is the side opposite γ .



Using the relationship

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

We get

$$\begin{aligned} c^2 &= 15.0^2 + 17.0^2 - 2(15.0)(17.0)\cos 89.0^\circ \\ &= 225 + 289 - 510(0.0175) \\ &= 505.10 \end{aligned}$$

So

$$c = 22.5$$

Now, since we know the measure of one angle and the length of the side opposite it, we can use the Law of Sines to complete the problem.

$$\frac{\sin 87.0^\circ}{22.1} = \frac{\sin \alpha}{15.0} \quad \text{and} \quad \frac{\sin 87.0^\circ}{22.1} = \frac{\sin \beta}{17.0}$$

This gives

$$\alpha = 42.7^\circ \quad \text{and} \quad \beta = 50.2^\circ$$

Note that due to round-off error, the angles do not add up to exactly 180° .

Three sides are known.

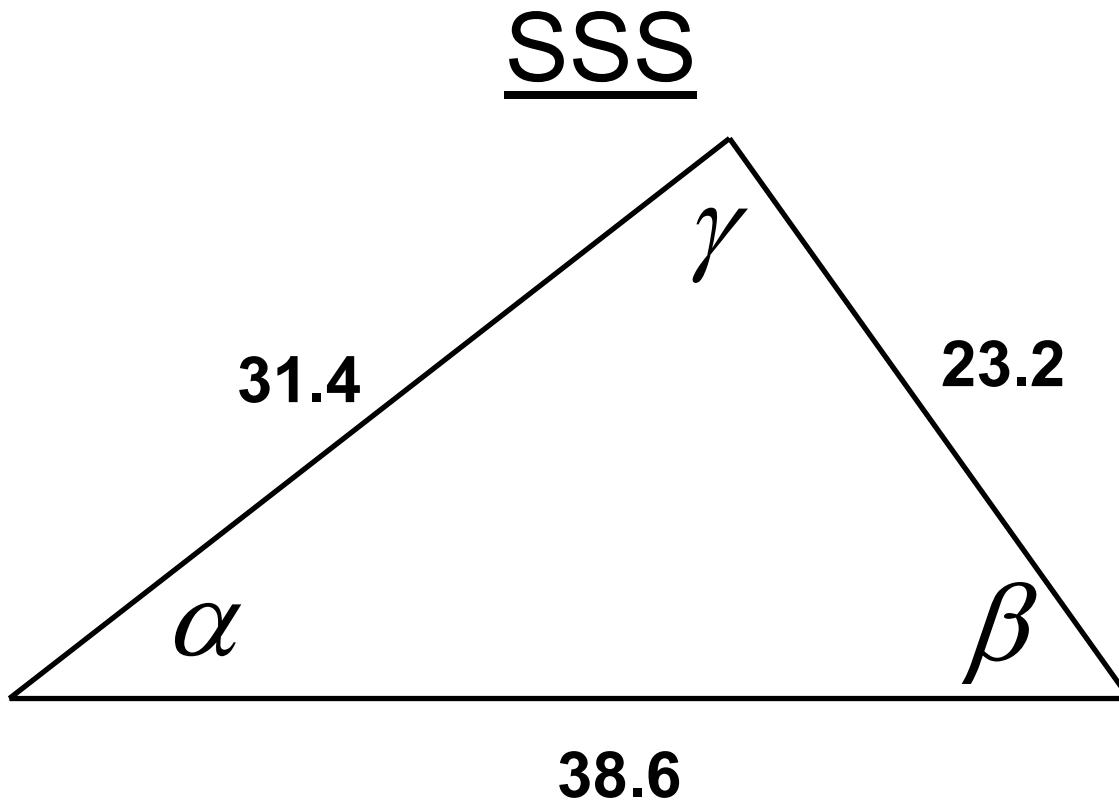
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In this figure, we need to find the three angles, α , β , and γ .

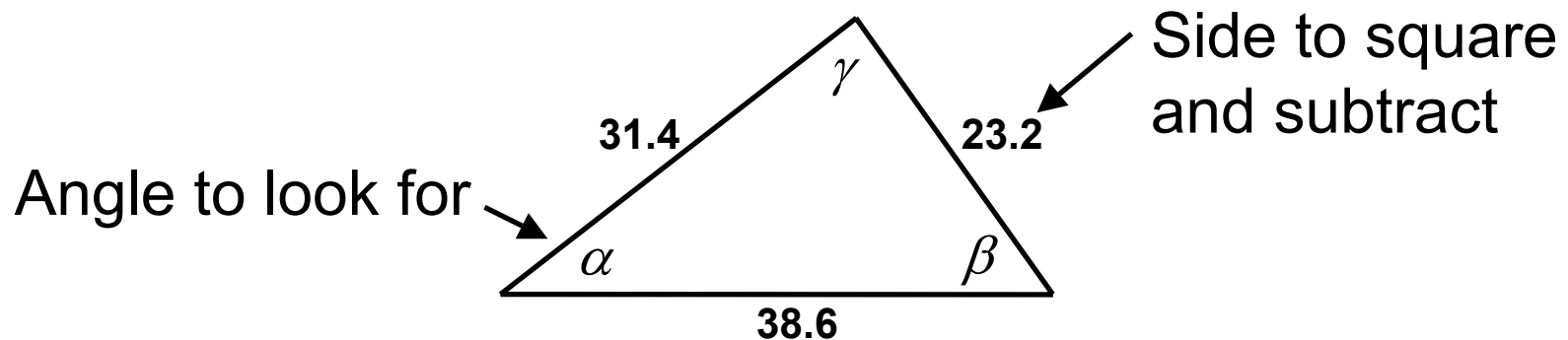
To solve a triangle when all three sides are known we must first find one angle using the Law of Cosines.

We must isolate and solve for the cosine of the angle we are seeking, then use the inverse cosine to find the angle.

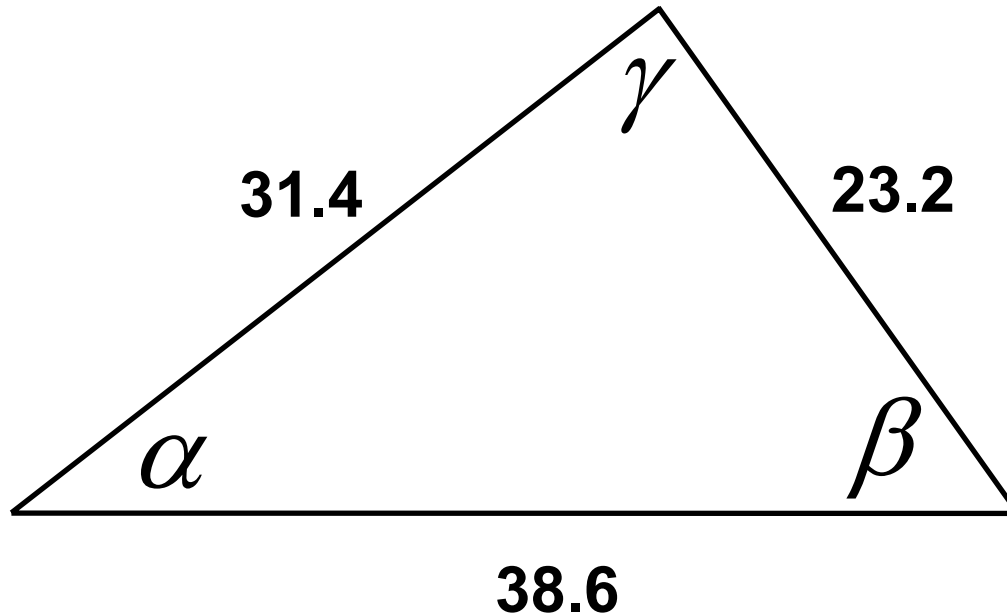
We do this by rewriting the Law of Cosines equation to the following form:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

In this form, the square being subtracted is the square of the side opposite the angle we are looking for.



We start by finding $\cos \alpha$.



$$\cos \alpha = \frac{31.4^2 + 38.6^2 - 23.2^2}{2(31.4)(38.6)}$$

From the equation

$$\cos \alpha = \frac{31.4^2 + 38.6^2 - 23.2^2}{2(31.4)(38.6)}$$

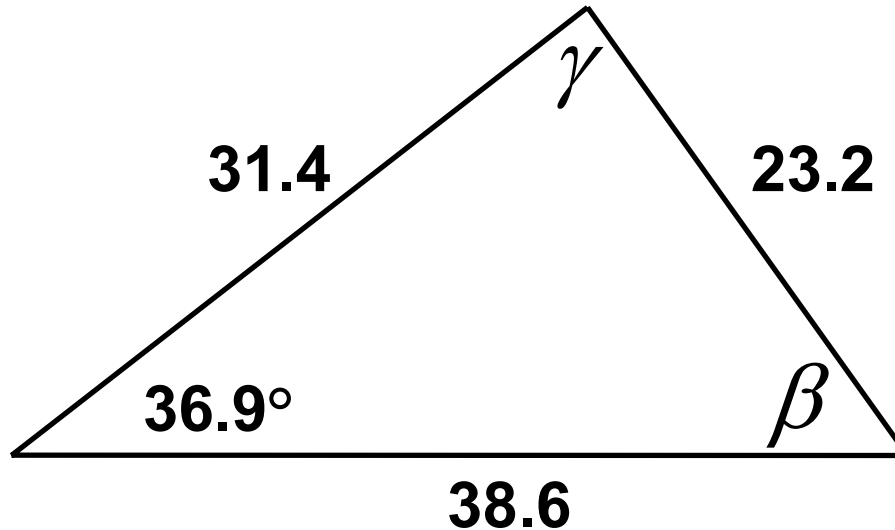
we get

$$\cos \alpha = 0.7993$$

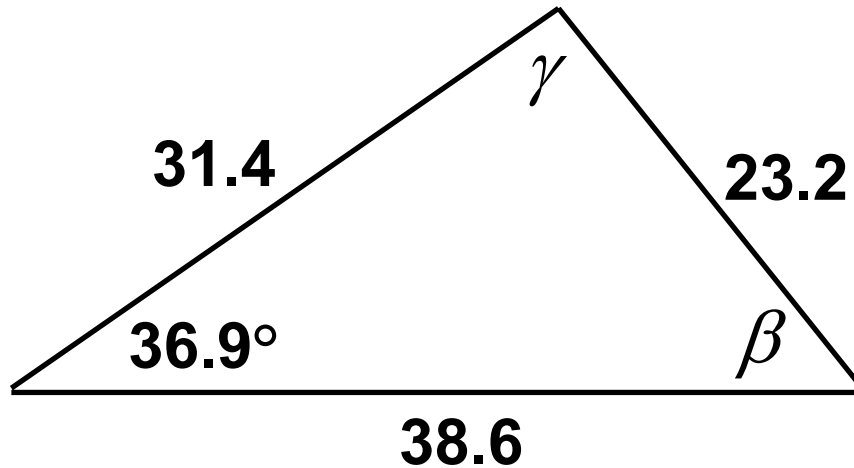
and

$$\alpha = 36.9^\circ$$

Our triangle now looks like this:



Again, since we have the measure for both a side and the angle opposite it, we can use the Law of Sines to complete the solution of this triangle.



Completing the solution we get the following:

$$\frac{\sin \beta}{31.4} = \frac{\sin 36.9^\circ}{23.2} \quad \text{and} \quad \frac{\sin \gamma}{38.6} = \frac{\sin 36.9^\circ}{23.2}$$

Solving these two equations we get the following:

$$\sin \beta = 0.8126$$

$$\sin \gamma = 0.9990$$

and

$$\beta = 54.4^\circ$$

$$\gamma = 87.3^\circ$$

Again, because of round-off error, the angles do not add up to exactly 180° .

Most of the round-off error can be avoided by storing the exact value you get for α and using that value to compute $\sin \alpha$.

Then, store $\sin \alpha$ in your calculator's memory also and use that value to get β and γ .

In this case we get the following:

$$\cos \alpha = .7993465562$$

$$\alpha = 36.9322517^\circ$$

$$\sin \alpha = .6008702714$$

$$\beta = 54.41437972^\circ$$

$$\gamma = 88.65337196$$

If we round off at this point we get $\alpha = 36.9^\circ$, $\beta = 54.4^\circ$ and $\gamma = 88.7^\circ$.

Now the three angles add up to 180° .

End of Law of Cosines

Title V

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